# Modeling of Manufacturing of Field-Effect Heterotransistors without P-n-junctions to Optimize Decreasing their Dimensions

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## Abstract:

It has been recently shown that manufacturing p-n-junctions, field-effect and bipolar transistors, thyristors in a multilayer structure by diffusion or ion implantation with the optimization of dopant and/or radiation defects leads to increase the sharpness of p-n-junctions (both single p-n-junctions and p-n-junctions framework their system). Due to the optimization, one can also obtain increasing of homogeneity of dopant in doped area. In this paper, we consider manufacturing of a field-effect heterotransistor without p-njunction. Framework the approach of manufacturing, we consider a heterostructure with specific configuration, doping required parts of the heterostructure by dopant diffusion or by ion implantation and optimization of annealing of dopant and/or radiation defects. The optimization gives us possibility to decrease dimensions of field-effect transistors. We introduce an analytical approach to model technological processes without crosslinking concentrations of dopant and radiation defects on interfaces between layers of heterostructure.

**Keyword:** Field-effect transistors, Decreasing of dimensions of transistors, Analytical approach to model transistors, Optimization of technological process.

# **1. INTRODUCTION**

The development of manufacturing of electronic devices causes necessary need to reduce the size of integrated circuit elements and their discrete analogs [1-7]. To reduce the required sizes of *p*-*n*-junctions and their systems (such transistors and thyristors) formed by diffusion and implantation, several approaches could be used. First of all, laser and microwave types of annealing should be considered [8-14]. By using these types of annealing, inhomogeneous distribution of

temperature is created. This gives us the possibility to increase sharpness of *p*-*n*-junctions with simultaneous increasing of homogeneity of distribution of dopant concentration in doped area [8-14]. In this situation, one can obtain more shallow *p*-*n*-junctions and at the same time decrease dimensions of transistors which include into itself the *p*-*n*-junctions. The second way to decrease dimensions of elements of integrated circuits is using of native inhomogeneity of heterostructure and optimizing the annealing of dopant and/or radiation defects [12-18]. In this case, one can obtain increasing of sharpness of *p*- *n*-junctions and at the same time increasing of homogeneity of distribution of dopant concentration in doped area [12-18]. Distribution of concentration of dopant could also change under the influence of radiation processing [19]. In this situation, radiation processing could also be used to increase the sharpness of single *p*-*n*-junction and *p*-*n*-junctions, which include into their system [20,21].

In this paper, we consider a heterostructure, which consists of a substrate and a multi-section epitaxial layer (Figures 1 and 2). Further, we consider manufacturing a field-effect transistor without p-n-junction in this heterostructure. Appropriate contacts are presented in the Figurs 1 and 2. A dopant has been infused or implanted in areas of source and drain before manufacturing the contacts to produce required types of conductivity. Also, we consider annealing of dopant and/or radiation defects to infuse the dopant on required depth (in the first case) or to decrease quantity of radiation defects gives us possibility to obtain

spreading of distribution of concentration of dopant. To restrict the dopant diffusion into substrate and to manufacture more thin structure, it is practicable to choose properties of heterostructure so that dopant diffusion coefficient in the substrate should be smaller than dopant diffusion coefficient in epitaxial layer as much as possible [12-18]. In this case, it is practicable to optimize annealing time [12-18]. If dopant does not achieve interface between layers of heterostructure during annealing of radiation defects, it is practicable to choose additional annealing of dopant. The main aims of the present paper are (i) modeling of redistribution of dopant and radiation defects and (ii) optimization of annealing time. Then, manufacturing thinner fieldeffect transistor based on using inhomogeneity of heterostructure is possible. It should be noted that the considered in our paper technological approach has not been recently described in literature (see, for example, [22-24] and similar works). In this situation, the described approach could be considered as a new approach.



Figure 1. Heterostructure with a substrate and multi-section epitaxial layer. View from top



Figure 2. Heterostructure with a substrate and multi-section epitaxial layer. View from one side

### 2. METHOD OF SOLUTION

To reach the goals of this study, we determine spatio-temporal distribution of concentration of dopant. We determine the distribution by solving the second Fick's law: [1-4]

$$\frac{\partial C(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_C \frac{\partial C(x, t)}{\partial x} \right]$$
(1)

With boundary and initial conditions

$$\frac{\partial C(x,t)}{\partial x}\Big|_{x=0} = 0, \quad \frac{\partial C(x,t)}{\partial x}\Big|_{x=L} = 0, \quad C(x,0) = fC(x).$$
(2)

Here C(x,t) is the spatio-temporal distribution of concentration of dopant; *T* is the temperature of annealing;  $D_C$  is the dopant diffusion coefficient. The value of dopant diffusion coefficient depends on properties of materials of heterostructure, speed of heating, and cooling of heterostructure (with account Arrhenius law). Dependences of dopant diffusion on parameters could be approximated by the following function [25-28]

$$D_{C} = D_{L}(x,T) \left[ 1 + \xi \frac{C^{\gamma}(x,t)}{P^{\gamma}(x,T)} \right] \left[ 1 + \varsigma_{1} \frac{V(x,t)}{V^{*}} + \varsigma_{2} \frac{V^{2}(x,t)}{(V^{*})^{2}} \right],$$
(3)

where  $D_L(x,T)$  is the approximation of dopant diffusion coefficient of coordinate (due to native in homogeneity of heterostructure) and temperature (due to Arrhenius law); P(x, T) is the limit of solubility of dopant; parameter  $\gamma$  depends on the properties of materials and could be an integer in the following interval  $\gamma \in [1,3]$  [28]; V(x,t) is the spatio-temporal distribution of concentration of radiation vacancies;  $V^*$  is the equilibrium distribution of Concentrational vacancies. dependence of dopant diffusion coefficient has been described in details by Yu Gotra in [28]. It should be noted that using doping of materials by diffusion leads to the absence of radiation damage  $\zeta_1 = \zeta_2 = 0$ . Spatio-temporal distributions of concentrations of radiation defects are determined by solving the following system of equations [26,27].

$$\begin{cases} \frac{\partial I(x,t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_{I}(x,T) \frac{\partial I(x,t)}{\partial x} \right] - k_{I,V}(x,T) I(x,t) V(x,t) - k_{I,I}(x,T) I^{2}(x,t) \\ \frac{\partial V(x,t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_{V}(x,T) \frac{\partial V(x,t)}{\partial x} \right] - k_{I,V}(x,T) I(x,t) V(x,t) - k_{V,V}(x,T) V^{2}(x,t) \end{cases}$$

(5a)

with initial condition:  $\rho(x,0)=f_{\rho}(x)$ And boundary conditions:

$$\frac{\partial I(x,t)}{\partial x}\bigg|_{x=0} = 0, \qquad \qquad \frac{\partial I(x,t)}{\partial x}\bigg|_{x=L} = 0,$$

Here  $\rho = I,V$ ; I(x,t) is the spatio-temporal distribution of concentration of interstitials;  $D_{\rho}(x,T)$  is the diffusion coefficient of point radiation defects (vacancies and interstitials); terms  $V^2(x,t)$  and  $I^2(x,t)$  corresponds to the generation of divacancies and diinterstitials;  $k_{I,V}(x,T)$ ,  $k_{I,I}(x,T)$  and  $k_{V,V}(x,T)$  are parameters of recombination of point radiation defects and generation their simplest

$$\frac{\partial V(x,t)}{\partial x}\bigg|_{x=0} = 0, \ \frac{\partial V(x,t)}{\partial x}\bigg|_{x=L} = 0,$$
  
I(x,0)=f<sub>1</sub>(x), V(x,0)=f<sub>V</sub>(x).  
(5b)

(4)

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complexes (divacancies and diinterstitials).

Here, spatio-temporal distributions of concentrations of divacancies  $\Phi_V(x,t)$  and diinterstitials  $\Phi_I(x,t)$  are determined by solving the following system of equations [26,27].

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$$\begin{cases}
\frac{\partial \Phi_{I}(x,t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_{\Phi I}(x,T) \frac{\partial \Phi_{I}(x,t)}{\partial x} \right] + k_{I,I}(x,T) I^{2}(x,t) - k_{I}(x,T) I(x,t) \\
\frac{\partial \Phi_{V}(x,t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_{\Phi V}(x,T) \frac{\partial \Phi_{V}(x,t)}{\partial x} \right] + k_{V,V}(x,T) V^{2}(x,t) - k_{V}(x,T) V(x,t)
\end{cases}$$
(6)

with boundary and initial conditions

$$\frac{\partial I(x,t)}{\partial x}\Big|_{x=0} = 0, \ \frac{\partial I(x,t)}{\partial x}\Big|_{x=L} = 0, \ \frac{\partial V(x,t)}{\partial x}\Big|_{x=0} = 0, \ \frac{\partial V(x,t)}{\partial x}\Big|_{x=L} = 0,$$

$$I(x,0)=f_{I}(x), V(x,0)=f_{V}(x).$$
(7)

Here  $D_{\phi l}(x,T)$  and  $D_{\phi V}(x,T)$  are diffusion coefficients of divacancies and diinterstitials;  $k_l(x,T)$  and  $k_V(x,T)$  are parameters of decay of complexes of radiation defects.

To determine spatio-temporal distributions of concentrations of point radiation defects, we used recently introduced approach [16,18]. To framework the approach, we transformed approximations of diffusion coefficients of point radiation defects to the following form:  $D_{\rho}(x,T) = D_{0\rho}[1 + \varepsilon_{\rho}g_{\rho}(x,T)]$ , where  $D_{0\rho}$  is the average value of the diffusion coefficients,  $0 \le \varepsilon_0 < 1$ ,  $|g_{\rho}(x,T)| \leq 1$ ,  $\rho = I, V$ . We also transformed parameters of recombination of point defects and generation of their complexes in the same form:  $k_{LV}(x, x)$   $T = k_{0I,V}[1 + \varepsilon_{I,V}g_{I,V}(x,T)], k_{I,I}(x,T) = k_{0I,I}[1 + \varepsilon_{I,I}g_{I,I}(x,T)]$ and  $k_{V,V}(x,T) = k_{0V,V}[1 + \varepsilon_{V,V}g_{V,V}(x, T)]$ , where  $k_{0\rho_{1},\rho_{2}}$ is the appropriate average values of the above parameters,  $0 \le \varepsilon_{LV} < 1$ ,  $0 \le \varepsilon_{LV} < 1$ ,  $0 \le \varepsilon_{VV} < 1$ ,  $|g_{LV}| < 1$  $(x,T) \le 1$ ,  $|g_{I,I}(x,T)| \le 1$ ,  $|g_{V,V}(x,T)| \le 1$ . We introduced following dimensionless the variables:  $\widetilde{I}(x,t) = I(x,t)/I^*, \quad \widetilde{V}(x,t) = V(x,t)/V^*, \quad \chi = x/L_x, \quad \eta$  $= y/L_y, \qquad \phi \qquad = z/L_z, \qquad \mathcal{G} = \sqrt{D_{0I}D_{0V}}t/L^2,$  $\omega = L^2 k_{0I,V} / \sqrt{D_{0I} D_{0V}}$ and  $\Omega_{o} = L^{2} k_{0,o,o} / \sqrt{D_{0l} D_{0l}}$ . The dimensionalization in Eqs.(4) and conditions (5) results in the following form:

$$\left| \frac{\partial \widetilde{I}(\chi, \theta)}{\partial \theta} = \frac{D_{0I}}{\sqrt{D_{0I}D_{0V}}} \frac{\partial}{\partial \chi} \left\{ \left[ 1 + \varepsilon_{I}g_{I}(\chi, T) \right] \frac{\partial \widetilde{I}(\chi, \theta)}{\partial \chi} \right\} - \Omega_{I} \left[ 1 + \varepsilon_{I,I}g_{I,I}(\chi, T) \right] \times \left[ X + \widetilde{I}^{2}(\chi, \theta) - \omega \left[ 1 + \varepsilon_{I,V}g_{I,V}(\chi, T) \right] \widetilde{I}(\chi, \theta) \widetilde{V}(\chi, \theta) \right] \right] \times \left[ \frac{\partial \widetilde{V}(\chi, \theta)}{\partial \theta} = \frac{D_{0V}}{\sqrt{D_{0I}D_{0V}}} \frac{\partial}{\partial \chi} \left\{ \left[ 1 + \varepsilon_{V}g_{V}(\chi, T) \right] \frac{\partial \widetilde{V}(\chi, \theta)}{\partial \chi} \right\} - \Omega_{V} \left[ 1 + \varepsilon_{V,V}g_{V,V}(\chi, T) \right] \right] \times \left[ \frac{\partial \widetilde{V}(\chi, \theta)}{\partial \chi} - \omega \left[ 1 + \varepsilon_{I,V}g_{I,V}(\chi, T) \right] \widetilde{I}(\chi, \theta) \widetilde{V}(\chi, \theta) \right] \right] \times \left[ \frac{\partial \widetilde{P}(\chi, \theta)}{\partial \chi} - \omega \left[ 1 + \varepsilon_{I,V}g_{I,V}(\chi, T) \right] \widetilde{I}(\chi, \theta) \widetilde{V}(\chi, \theta) \right] \right] + \left[ \frac{\partial \widetilde{P}(\chi, \theta)}{\partial \chi} - \omega \left[ 1 + \varepsilon_{I,V}g_{I,V}(\chi, T) \right] = 0, \quad \widetilde{P}(\chi, \theta) = \frac{f_{\rho}(\chi, \theta)}{\rho^{*}}. \right]$$

We determine solutions of Eqs.(8) and conditions (9) framework recently introduce [16,18]. To framework the approach, we determine the solutions as the following power series

 $\widetilde{\rho}(\chi, \mathcal{G}) = \sum_{i=0}^{\infty} \varepsilon_{\rho}^{i} \sum_{j=0}^{\infty} \omega^{j} \sum_{k=0}^{\infty} \Omega_{\rho}^{k} \widetilde{\rho}_{ijk}(\chi, \mathcal{G})$ (10)

Substitution of the series Eq. (10) into Eqs.(8) and conditions (9) gives us the possibility

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to obtain equations for the zero-order approximations of concentrations of point defects  $\widetilde{I}_{_{000}}(\chi, \mathcal{G})$  and  $\widetilde{V}_{_{000}}(\chi, \mathcal{G})$ , corrections are  $\widetilde{I}_{_{ijk}}(\chi, \mathcal{G})$  and  $\widetilde{V}_{_{ijk}}(\chi, \mathcal{G})$   $(i \ge 1, j \ge 1, k \ge 1)$  and

conditions for all functions are  $\widetilde{T}_{ijk}(\chi, \mathcal{G})$  and  $\widetilde{V}_{ijk}(\chi, \mathcal{G})$   $(i \ge 0, j \ge 0, k \ge 0)$ .

$$\begin{cases} \frac{\partial \tilde{I}_{000}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial^2 \tilde{I}_{000}(\chi, \vartheta)}{\partial \chi^2} \\ \frac{\partial \tilde{V}_{000}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial^2 \tilde{V}_{000}(\chi, \vartheta)}{\partial \chi^2}; \end{cases}$$

$$\begin{cases} \frac{\partial \widetilde{I}_{i00}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial^2 \widetilde{I}_{i00}(\chi, \vartheta)}{\partial \chi^2} + \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial}{\partial \chi} \left[ g_I(\chi, T) \frac{\partial \widetilde{I}_{i-100}(\chi, \vartheta)}{\partial \chi} \right] \\ \frac{\partial \widetilde{V}_{i00}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial^2 \widetilde{V}_{i00}(\chi, \vartheta)}{\partial \chi^2} + \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial}{\partial \chi} \left[ g_V(\chi, T) \frac{\partial \widetilde{V}_{i-100}(\chi, \vartheta)}{\partial \chi} \right], i \ge 1; \end{cases}$$

$$\begin{cases} \frac{\partial \tilde{I}_{010}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial^2 \tilde{I}_{010}(\chi, \vartheta)}{\partial \chi^2} - \left[1 + \varepsilon_{I,V} g_{I,V}(\chi, T)\right] \tilde{I}_{000}(\chi, \vartheta) \tilde{V}_{000}(\chi, \vartheta) \\ \frac{\partial \tilde{V}_{010}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial^2 \tilde{V}_{010}(\chi, \vartheta)}{\partial \chi^2} - \left[1 + \varepsilon_{I,V} g_{I,V}(\chi, T)\right] \tilde{I}_{000}(\chi, \vartheta) \tilde{V}_{000}(\chi, \vartheta) \end{cases};$$

$$\begin{cases} \frac{\partial \tilde{I}_{020}(\chi, \theta)}{\partial \theta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial^2 \tilde{I}_{020}(\chi, \theta)}{\partial \chi^2} - \\ -\left[1 + \varepsilon_{I,V} g_{I,V}(\chi, T)\right] \left[\tilde{I}_{010}(\chi, \theta) \tilde{V}_{000}(\chi, \theta) + \tilde{I}_{000}(\chi, \theta) \tilde{V}_{010}(\chi, \theta)\right]; \\ \frac{\partial \tilde{V}_{020}(\chi, \theta)}{\partial \theta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial^2 \tilde{V}_{020}(\chi, \theta)}{\partial \chi^2} - \\ -\left[1 + \varepsilon_{I,V} g_{I,V}(\chi, T)\right] \left[\tilde{I}_{010}(\chi, \theta) \tilde{V}_{000}(\chi, \theta) + \tilde{I}_{000}(\chi, \theta) \tilde{V}_{010}(\chi, \theta)\right] \\ \end{cases}$$

$$\begin{cases} \frac{\partial I_{001}(\chi, \mathcal{G})}{\partial \mathcal{G}} = \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial I_{001}(\chi, \mathcal{G})}{\partial \chi^2} - \left[1 + \varepsilon_{I,I} g_{I,I}(\chi, T)\right] \tilde{I}_{000}^2(\chi, \mathcal{G}) \\ \frac{\partial \tilde{V}_{001}(\chi, \mathcal{G})}{\partial \mathcal{G}} = \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial^2 \tilde{V}_{001}(\chi, \mathcal{G})}{\partial \chi^2} - \left[1 + \varepsilon_{I,I} g_{I,I}(\chi, T)\right] \tilde{V}_{000}^2(\chi, \mathcal{G}) \end{cases};$$

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$$\begin{cases} \frac{\partial \tilde{I}_{110}(\chi, 9)}{\partial 9} = \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial^2 \tilde{I}_{110}(\chi, 9)}{\partial \chi^2} + \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial}{\partial \chi} \left[ g_I(\chi, T) \frac{\partial \tilde{I}_{010}(\chi, 9)}{\partial \chi} \right] - \\ - \left[ 1 + \varepsilon_{I,I} g_{I,I}(\chi, T) \right] \left[ \tilde{I}_{100}(\chi, 9) \tilde{V}_{000}(\chi, 9) + \tilde{I}_{000}(\chi, 9) \tilde{V}_{100}(\chi, 9) \right] \\ \frac{\partial \tilde{V}_{110}(\chi, 9)}{\partial 9} = \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial^2 \tilde{V}_{110}(\chi, 9)}{\partial \chi^2} + \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial}{\partial \chi} \left[ g_V(\chi, T) \frac{\partial \tilde{V}_{010}(\chi, 9)}{\partial \chi} \right] - \\ - \left[ 1 + \varepsilon_{V,V} g_{V,V}(\chi, T) \right] \left[ \tilde{V}_{100}(\chi, 9) \tilde{I}_{000}(\chi, 9) + \tilde{V}_{000}(\chi, 9) \tilde{I}_{100}(\chi, 9) \right] \end{cases}$$

$$\begin{cases} \frac{\partial \tilde{I}_{002}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial^2 \tilde{I}_{002}(\chi, \vartheta)}{\partial \chi^2} - \left[1 + \varepsilon_{I,I} g_{I,I}(\chi, T)\right] \tilde{I}_{001}(\chi, \vartheta) \tilde{I}_{000}(\chi, \vartheta) \\ \frac{\partial \tilde{V}_{002}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial^2 \tilde{V}_{002}(\chi, \vartheta)}{\partial \chi^2} - \left[1 + \varepsilon_{I,I} g_{I,I}(\chi, T)\right] \tilde{V}_{001}(\chi, \vartheta) \tilde{V}_{000}(\chi, \vartheta) \\ ; \end{cases}$$

$$\begin{cases} \frac{\partial \tilde{I}_{101}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial^2 \tilde{I}_{101}(\chi, \vartheta)}{\partial \chi^2} + \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial}{\partial \chi} \left[ g_I(\chi, T) \frac{\partial \tilde{I}_{001}(\chi, \vartheta)}{\partial \chi} \right] - \\ - \left[ 1 + \varepsilon_I g_I(\chi, T) \right] \tilde{I}_{100}(\chi, \vartheta) \tilde{V}_{000}(\chi, \vartheta) \\ \frac{\partial \tilde{V}_{101}(\chi, \vartheta)}{\partial \vartheta} = \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial^2 \tilde{V}_{101}(\chi, \vartheta)}{\partial \chi^2} + \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial}{\partial \chi} \left[ g_V(\chi, T) \frac{\partial \tilde{V}_{001}(\chi, \vartheta)}{\partial \chi} \right] - \\ - \left[ 1 + \varepsilon_V g_V(\chi, T) \right] \tilde{I}_{000}(\chi, \vartheta) \tilde{V}_{100}(\chi, \vartheta) \end{cases};$$

$$\begin{cases} \frac{\partial \tilde{I}_{011}(\chi, 9)}{\partial 9} = \sqrt{\frac{D_{0I}}{D_{0V}}} \frac{\partial^2 \tilde{I}_{011}(\chi, 9)}{\partial \chi^2} - \left[1 + \varepsilon_{I,I}g_{I,I}(\chi, T)\right] \tilde{I}_{000}(\chi, 9)\tilde{I}_{010}(\chi, 9) - \left[1 + \varepsilon_{I,V}g_{I,V}(\chi, T)\right] \tilde{I}_{001}(\chi, 9)\tilde{V}_{000}(\chi, 9) \\ - \left[1 + \varepsilon_{I,V}g_{I,V}(\chi, T)\right] \tilde{I}_{001}(\chi, 9)\tilde{V}_{000}(\chi, 9) \\ \frac{\partial \tilde{V}_{011}(\chi, 9)}{\partial 9} = \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial^2 \tilde{V}_{011}(\chi, 9)}{\partial \chi^2} - \left[1 + \varepsilon_{V,V}g_{V,V}(\chi, T)\right] \tilde{V}_{000}(\chi, 9)\tilde{V}_{010}(\chi, 9) - \left[1 + \varepsilon_{I,V}g_{I,V}(\chi, T)\right] \tilde{I}_{000}(\chi, 9)\tilde{V}_{001}(\chi, 9) \end{cases}$$

$$\left. \frac{\partial \widetilde{\rho}_{ijk}(\chi, \vartheta)}{\partial \chi} \right|_{x=0} = 0, \left. \frac{\partial \widetilde{\rho}_{ijk}(\chi, \vartheta)}{\partial \chi} \right|_{x=1} = 0, (i \ge 0, j \ge 0, k \ge 0);$$
  
 $\widetilde{\rho}_{000}(\chi, 0) = f_{\rho}(\chi) / \rho^*, \ \widetilde{\rho}_{ijk}(\chi, 0) = 0 \ (i \ge 1, j \ge 1, k \ge 1).$ 

We obtained solutions of the obtained equations by the Fourier approach [29,30]. The

solutions considering appropriate boundary and initial conditions could be written as

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where  $s_n(\chi) = sin(\pi n \chi)$ ;

$$\begin{split} \tilde{\rho}_{010}(\chi, \vartheta) &= -2\sum_{n=1}^{\infty} c_n(\chi, \phi) e_{n\rho}(\vartheta) \int_{0}^{\vartheta} e_{n\rho}(-\tau) \int_{0}^{1} c(u) [1 + \varepsilon_{I,V} g_{I,V}(u,T)] I_{000}(u,\tau) V_{000}(u,\tau) du d\tau; \\ \tilde{\rho}_{020}(\chi, \vartheta) &= -2\sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} c_n(\chi) e_{n\rho}(\vartheta) \int_{0}^{\vartheta} e_{n\rho}(-\tau) \int_{0}^{1} c(u) [1 + \varepsilon_{I,V} g_{I,V}(u,T)] \times \\ &\times [\tilde{I}_{010}(\chi, \vartheta) \tilde{V}_{000}(\chi, \vartheta) + \tilde{I}_{000}(\chi, \vartheta) \tilde{V}_{010}(\chi, \vartheta)] du d\tau; \\ \\ \left\{ \tilde{I}_{110}(\chi, \vartheta) &= -2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n c_n(\chi) e_{nI}(\vartheta) \int_{0}^{\vartheta} e_{nI}(-\tau) \int_{0}^{1} s_n(u) g_I(u,T) \frac{\partial \tilde{I}_{010}(u,\tau)}{\partial u} du d\tau - \\ &- 2\sum_{n=1}^{\infty} c_n(\chi) e_{nI}(\vartheta) \int_{0}^{\vartheta} e_{nI}(-\tau) \int_{0}^{1} c_n(u) [\tilde{I}_{100}(u,\tau) \tilde{V}_{000}(u,\tau) + \tilde{I}_{000}(u,\tau) \tilde{V}_{100}(u,\tau)] \times \\ &\times [1 + \varepsilon_{I,V} g_{I,V}(u,T)] du d\tau \\ &\times [1 + \varepsilon_{I,V} g_{I,V}(u,T)] du d\tau ; \\ &\tilde{V}_{110}(\chi, \vartheta) &= -2\pi \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n c_n(\chi) e_{nV}(\vartheta) \int_{0}^{\vartheta} e_{nV}(-\tau) \int_{0}^{1} s_n(u) g_V(u,T) \frac{\partial \tilde{V}_{010}(u,\tau)}{\partial u} du d\tau - \\ &= 2\sum_{n=1}^{\infty} c_n(\chi) e_n(\vartheta) \int_{0}^{\vartheta} e_n(-\tau) \int_{0}^{1} c_n(u) [\tilde{I}_{-v}(u,\tau) \tilde{V}_{-v}(u,\tau) + \tilde{I}_{-v}(u,\tau) \tilde{V}_{-v}(u,\tau)] \times \\ &\times [1 + \varepsilon_{I,V} g_{I,V}(u,T)] du d\tau ; \end{split}$$

$$-2\sum_{n=1}^{\infty}c_{n}(\chi)e_{nV}(\mathcal{G})\int_{0}e_{nV}(-\tau)\int_{0}c_{n}(u)[\widetilde{I}_{000}(u,\tau)\widetilde{V}_{100}(u,\tau)+\widetilde{I}_{100}(u,\tau)\widetilde{V}_{000}(u,\tau)]\times$$

$$\times [1+\varepsilon_{I,V}g_{I,V}(u,T)]dud\tau$$

$$\widetilde{\mathcal{P}}_{001}(\chi, \mathcal{G}) = -2\sum_{n=1}^{\infty} c_n(\chi) e_{n\rho}(\mathcal{G}) \int_0^{\mathcal{G}} e_{n\rho}(-\tau) \int_0^1 c_n(u) \left[1 + \varepsilon_{\rho,\rho} g_{\rho,\rho}(u,T)\right] \widetilde{\mathcal{P}}_{000}^2(u,\tau) du d\tau;$$

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$$\mathcal{P}_{002}(\chi, 9) = -2\sum_{n=1}^{\infty} c_n(\chi) e_{n\rho}(9) \int_{0}^{9} e_{n\rho}(-\tau) \int_{0}^{1} c_n(u) [1 + \varepsilon_{\rho,\rho} g_{\rho,\rho}(u,T)] \mathcal{P}_{001}(u,\tau) \mathcal{P}_{000}(u,\tau) \, du \, d\tau;$$

$$\begin{cases} \tilde{I}_{101}(\chi, 9) = -2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n c_n(\chi) e_{nI}(9) \int_{0}^{9} e_{nI}(-\tau) \int_{0}^{1} s_n(u) g_I(u,T) \frac{\partial \tilde{I}_{001}(u,\tau)}{\partial u} \, du \, d\tau - \\ -2\sum_{n=1}^{\infty} c_n(\chi) e_{nI}(9) \int_{0}^{9} e_{nI}(-\tau) \int_{0}^{1} c_n(u) [1 + \varepsilon_{I,V} g_{I,V}(u,T)] \tilde{I}_{100}(u,\tau) \mathcal{V}_{000}(u,\tau) \, du \, d\tau \\ \tilde{V}_{101}(\chi, 9) = -2\pi \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n c_n(\chi) e_{nV}(9) \int_{0}^{9} e_{nV}(-\tau) \int_{0}^{1} s_n(u) g_V(u,T) \frac{\partial \tilde{V}_{001}(u,\tau)}{\partial u} \, du \, d\tau - \\ -2\sum_{n=1}^{\infty} c_n(\chi) e_{nV}(9) \int_{0}^{9} e_{nV}(-\tau) \int_{0}^{1} c_n(u) [1 + \varepsilon_{I,V} g_{I,V}(u,T)] \tilde{I}_{000}(u,\tau) \mathcal{V}_{100}(u,\tau) \, du \, d\tau - \\ -2\sum_{n=1}^{\infty} c_n(\chi) e_{nV}(9) \int_{0}^{9} e_{nV}(-\tau) \int_{0}^{1} c_n(u) [1 + \varepsilon_{I,V} g_{I,V}(u,T)] \tilde{I}_{000}(u,\tau) \mathcal{V}_{100}(u,\tau) \, du \, d\tau - \\ -2\sum_{n=1}^{\infty} c_n(\chi) e_{nV}(9) \int_{0}^{9} e_{nV}(-\tau) \int_{0}^{1} c_n(u) [1 + \varepsilon_{I,V} g_{I,V}(u,T)] \tilde{I}_{000}(u,\tau) \mathcal{V}_{100}(u,\tau) \, du \, d\tau - \\ -2\sum_{n=1}^{\infty} c_n(\chi) e_{nV}(9) \int_{0}^{9} e_{nV}(-\tau) \int_{0}^{1} c_n(u) [1 + \varepsilon_{I,V} g_{I,V}(u,T)] \tilde{I}_{000}(u,\tau) \mathcal{V}_{100}(u,\tau) \, du \, d\tau - \\ -2\sum_{n=1}^{\infty} c_n(\chi) e_{nV}(9) \int_{0}^{9} e_{nV}(-\tau) \int_{0}^{1} c_n(u) [1 + \varepsilon_{I,V} g_{I,V}(u,T)] \tilde{I}_{000}(u,\tau) \mathcal{V}_{100}(u,\tau) \, du \, d\tau - \\ -2\sum_{n=1}^{\infty} c_n(\chi) e_{nV}(9) \int_{0}^{9} e_{nV}(-\tau) \int_{0}^{1} c_n(u) [1 + \varepsilon_{I,V} g_{I,V}(u,T)] \tilde{I}_{000}(u,\tau) \, du \, d\tau - \\ -2\sum_{n=1}^{\infty} c_n(\chi) e_{nV}(9) \int_{0}^{9} e_{nV}(-\tau) \int_{0}^{1} c_n(u) [1 + \varepsilon_{I,V} g_{I,V}(u,T)] \tilde{I}_{000}(u,\tau) \, du \, d\tau - \\ -2\sum_{n=1}^{\infty} c_n(\chi) e_{nV}(9) \int_{0}^{9} e_{nV}(-\tau) \int_{0}^{1} c_n(u) [1 + \varepsilon_{I,V} g_{I,V}(u,T)] \tilde{I}_{000}(u,\tau) \, du \, d\tau - \\ -2\sum_{n=1}^{\infty} c_n(\chi) e_{nV}(9) \int_{0}^{9} e_{nV}(-\tau) \int_{0}^{1} c_n(\chi) e_{nV}(\chi) \, du \, d\tau - \\ -2\sum_{n=1}^{\infty} c_n(\chi) e_{nV}(\eta) \int_{0}^{9} e_{nV}(-\tau) \int_{0}^{1} c_n(\chi) e_{nV}(\chi) \, du \, d\tau - \\ -2\sum_{n=1}^{\infty} c_n(\chi) e_{nV}(\eta) \int_{0}^{9} e_{nV}(-\tau) \int_{0}^{1} c_n(\chi) e_{nV}(\eta) \, du \, d\tau - \\ -2\sum_{n=1}^{\infty} c_n(\chi) e_{nV}(\eta) \int_{0}^{9} e_{nV}(\eta)$$

$$\begin{cases} \widetilde{I}_{011}(\chi, \vartheta) = -2\sum_{n=1}^{\infty} c_n(\chi) e_{nI}(\vartheta) \int_{0}^{\vartheta} e_{nI}(-\tau) \int_{0}^{1} c_n(u) \left\{ \left[ 1 + \varepsilon_{I,I} g_{I,I}(u,T) \right] \widetilde{I}_{000}(u,\tau) \widetilde{I}_{010}(u,\tau) + \left[ 1 + \varepsilon_{I,V} g_{I,V}(u,T) \right] \widetilde{I}_{001}(u,\tau) \widetilde{V}_{000}(u,\tau) \right\} du d\tau \\ \widetilde{V}_{011}(\chi, \vartheta) = -2\sum_{n=1}^{\infty} c_n(\chi) e_{nV}(\vartheta) \int_{0}^{\vartheta} e_{nV}(-\tau) \int_{0}^{1} c_n(u) \left\{ \left[ 1 + \varepsilon_{V,V} g_{V,V}(u,T) \right] \widetilde{V}_{000}(u,\tau) \widetilde{V}_{010}(u,\tau) + \left[ 1 + \varepsilon_{I,V} g_{I,V}(u,T) \right] \widetilde{I}_{001}(u,\tau) \widetilde{V}_{000}(u,\tau) \right\} du d\tau. \end{cases}$$

Further, we determined spatio-temporal distribution of concentration of complexes of point radiation defects. To calculate the distribution, we transformed the approximation of diffusion coefficient to the following form:

 $D_{\phi\rho}(x,T)=D_{0\phi\rho}[1+\varepsilon_{\phi\rho}g_{\phi\rho}(x,T)]$ , where  $D_{0\phi\rho}$  are the average values of diffusion coefficient. After this transformation, Eq.(6) transforms to the following form:

$$\begin{cases} \frac{\partial \Phi_{I}(x,t)}{\partial t} = D_{0\Phi I} \frac{\partial}{\partial x} \left\{ \left[ 1 + \varepsilon_{\Phi I} g_{\Phi I}(x,T) \right] \frac{\partial \Phi_{I}(x,t)}{\partial x} \right\} + k_{I,I}(x,T) I^{2}(x,t) - k_{I}(x,T) I(x,t) \\ \frac{\partial \Phi_{V}(x,t)}{\partial t} = D_{0\Phi V} \frac{\partial}{\partial x} \left\{ \left[ 1 + \varepsilon_{\Phi V} g_{\Phi V}(x,T) \right] \frac{\partial \Phi_{V}(x,t)}{\partial x} \right\} + k_{V,V}(x,T) V^{2}(x,t) - k_{V}(x,T) V(x,t) \end{cases}$$

Let us determine solutions of the above equations as the following power series

$$\Phi_{\rho}(x,t) = \sum_{i=0}^{\infty} \varepsilon_{\Phi\rho}^{i} \Phi_{\rho i}(x,t).$$
(11)

Substitution of the series Eq. (11) into Eq.(6) and considering their conditions gives us the

possibility to obtain equations for initial-order approximations of concentrations of complexes, their corrections and conditions for all equations in the following form:

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$$\begin{cases} \frac{\partial \Phi_{I0}(x,t)}{\partial t} = D_{0\Phi I} \frac{\partial^2 \Phi_{I0}(x,t)}{\partial x^2} + k_{I,I}(x,T)I^2(x,t) - k_I(x,T)I(x,t) \\ \frac{\partial \Phi_{V0}(x,t)}{\partial t} = D_{0\Phi V} \frac{\partial^2 \Phi_{V0}(x,t)}{\partial x^2} + k_{V,V}(x,T)V^2(x,t) - k_V(x,T)V(x,t) \end{cases}; \\ \begin{cases} \frac{\partial \Phi_{Ii}(x,t)}{\partial t} = D_{0\Phi I} \frac{\partial^2 \Phi_{Ii}(x,t)}{\partial x^2} + D_{0\Phi I} \frac{\partial}{\partial x} \left[ g_{\Phi I}(x,T) \frac{\partial \Phi_{Ii-1}(x,t)}{\partial x} \right] \\ \frac{\partial \Phi_{Vi}(x,t)}{\partial t} = D_{0\Phi V} \frac{\partial^2 \Phi_{Vi}(x,t)}{\partial x^2} + D_{0\Phi V} \frac{\partial}{\partial x} \left[ g_{\Phi V}(x,T) \frac{\partial \Phi_{Vi-1}(x,t)}{\partial x} \right], i \ge 1; \\ \frac{\partial \Phi_{\rho i}(x,t)}{\partial x} \bigg|_{x=0} = 0, \quad \frac{\partial \Phi_{\rho i}(x,t)}{\partial x} \bigg|_{x=L} = 0, \quad i \ge 0; \quad \Phi_{\rho 0}(x,0) = f_{\Phi \rho}(x), \quad \Phi_{\rho i}(x,0) = 0, \quad i \ge 1 \end{cases}$$

Solutions of the above equations have been obtained by using the Fourier approach [29,30] and

could be written as

$$\Phi_{\rho 0}(x,t) = \frac{1}{L} + \frac{2}{L} \sum_{n=1}^{\infty} F_{n \Phi_{\rho}} c(x) e_{n \Phi_{\rho}}(t) + \\ + \frac{2}{L} \sum_{n=1}^{\infty} n c_{n}(x) e_{\Phi_{\rho} n}(t) \int_{0}^{t} e_{\Phi_{\rho} n}(-\tau) \int_{0}^{L} c_{n}(u) [k_{I,I}(u,T)I^{2}(u,\tau) - k_{I}(u,T)I(u,\tau)] du d\tau , \\ \text{where } e_{n \Phi_{\rho}}(t) = exp(-\pi^{2}n^{2}D_{0\Phi_{\rho}}t/L^{2}), \ F_{n \Phi_{\rho}} = \int_{0}^{L} c_{n}(u) f_{\Phi_{\rho}}(u) du , \ c_{n}(L) = cos(\pi n x/L); \\ \Phi_{\rho i}(x,t) = -\frac{2\pi}{L^{2}} \sum_{n=1}^{\infty} n c_{n}(x) e_{\Phi_{\rho} n}(t) \int_{0}^{t} e_{\Phi_{\rho} n}(-\tau) \int_{0}^{L} s_{n}(u) g_{\Phi_{\rho}}(u,T) \frac{\partial \Phi_{I_{\rho} i-1}(u,\tau)}{\partial u} du d\tau , i \ge 1, \end{cases}$$

where  $S_n(L) = \sin (\pi n x/L)$ .

Spatio-temporal distribution of concentration of dopant was determined by using the recently introduced approach. To framework the approach, it is necessary to transform approximation of dopant diffusion coefficient to the following form:  $D_L(x,T) = D_{0L}[1+\varepsilon_L g_L(x,T)]$ , where  $D_{0L}$  is the average value of dopant diffusion coefficient,  $0 \le \varepsilon_L < 1$ ,  $|g_L(x,T)| \le 1$ . Further, we determined solution of the Eq. (1) as the following power series:

$$C(x,t) = \sum_{i=0}^{\infty} \varepsilon_L^i \sum_{j=1}^{\infty} \zeta^j C_{ij}(x,t).$$

Substitution of the series into Eq. (1) and conditions (2) gives us possibility to obtain zeroorder approximation of dopant concentration  $C_{00}(x,t)$ , corrections  $C_{ij}(x,t)$   $(i \ge 1, j \ge 1)$  and conditions for all functions. The equations and conditions could be written as

$$\frac{\partial C_{00}(x,t)}{\partial t} = D_{0L} \frac{\partial^2 C_{00}(x,t)}{\partial x^2};$$
  
$$\frac{\partial C_{i0}(x,t)}{\partial t} = D_{0L} \frac{\partial^2 C_{i0}(x,t)}{\partial x^2} + D_{0L} \frac{\partial}{\partial x} \left[ g_L(x,T) \frac{\partial C_{i-10}(x,t)}{\partial x} \right], i \ge 1;$$

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$$\begin{split} \frac{\partial C_{01}(x,t)}{\partial t} &= D_{0L} \frac{\partial^2 C_{01}(x,t)}{\partial x^2} + D_{0L} \frac{\partial}{\partial x} \left[ \frac{C_{00}^{\gamma}(x,t)}{P^{\gamma}(x,T)} \frac{\partial C_{00}(x,t)}{\partial x} \right]; \\ \frac{\partial C_{02}(x,t)}{\partial t} &= D_{0L} \frac{\partial^2 C_{02}(x,t)}{\partial x^2} + D_{0L} \frac{\partial}{\partial x} \left[ C_{01}(x,t) \frac{C_{00}^{\gamma-1}(x,t)}{P^{\gamma}(x,T)} \frac{\partial C_{00}(x,t)}{\partial x} \right] + \\ &+ D_{0L} \frac{\partial}{\partial x} \left[ \frac{C_{00}^{\gamma}(x,t)}{P^{\gamma}(x,T)} \frac{\partial C_{01}(x,t)}{\partial x} \right]; \\ \frac{\partial C_{11}(x,t)}{\partial t} &= D_{0L} \frac{\partial^2 C_{11}(x,t)}{\partial x^2} + D_{0L} \frac{\partial}{\partial x} \left[ C_{10}(x,t) \frac{C_{00}^{\gamma-1}(x,t)}{P^{\gamma}(x,T)} \frac{\partial C_{00}(x,t)}{\partial x} \right] + \\ &+ D_{0L} \frac{\partial}{\partial x} \left[ \frac{C_{00}^{\gamma}(x,t)}{P^{\gamma}(x,T)} \frac{\partial C_{10}(x,t)}{\partial x} \right] + D_{0L} \frac{\partial}{\partial x} \left[ g_L(x,T) \frac{\partial C_{01}(x,t)}{\partial x} \right]; \\ \frac{\partial C_{1j}(x,t)}{\partial x} \bigg|_{x=0} &= 0, \quad \frac{\partial C_{1j}(x,t)}{\partial x} \bigg|_{x=L} = 0, \quad i \ge 0; \quad C_{00}(x,0) = f_{C}(x), \quad C_{1j}(x,0) = 0, \quad i \ge I, \quad j \ge I. \end{split}$$

Solutions of the equations considering appropriate boundary and initial conditions have

been calculated by using the Fourier approach [29,30]. The solutions could be written as:

$$\begin{split} C_{00}(x,t) &= \frac{1}{L} + \frac{2}{L} \sum_{n=1}^{\infty} F_{nc} c(x) e_{nc}(t), \\ \text{where } e_{nc}(t) &= exp\left(-\pi^{2} n^{2} D_{0c} t/L^{2}\right), \ F_{nc} = \int_{0}^{L} c_{n}(u) f_{c}(u) du ; \\ C_{i0}(x,t) &= -\frac{2\pi}{L^{2}} \sum_{n=1}^{\infty} n \ F_{nc} c_{n}(x) e_{nc}(t) \int_{0}^{t} e_{nc}(-\tau) \int_{0}^{L} s_{n}(u) g_{L}(u,T) \frac{\partial C_{i-10}(u,\tau)}{\partial u} du d\tau, i \ge 1; \\ C_{01}(x,t) &= -\frac{2\pi}{L^{2}} \sum_{n=1}^{\infty} n \ F_{nc} c_{n}(x) e_{nc}(t) \int_{0}^{t} e_{nc}(-\tau) \int_{0}^{t} s_{n}(u) \frac{C_{00}^{\prime}(u,\tau)}{P^{\prime}(u,T)} \frac{\partial C_{00}(u,\tau)}{\partial u} du d\tau; \\ C_{02}(x,t) &= -\frac{2\pi}{L^{2}} \sum_{n=1}^{\infty} n \ F_{nc} c_{n}(x) e_{nc}(t) \int_{0}^{t} e_{nc}(-\tau) \int_{0}^{t} s_{n}(u) C_{01}(u,\tau) \frac{C_{00}^{\prime \prime}(u,\tau)}{P^{\prime}(u,T)} \frac{\partial C_{00}(u,\tau)}{\partial u} du d\tau; \\ C_{02}(x,t) &= -\frac{2\pi}{L^{2}} \sum_{n=1}^{\infty} n \ F_{nc} c_{n}(x) e_{nc}(t) \int_{0}^{t} e_{nc}(-\tau) \int_{0}^{t} s_{n}(u) C_{01}(u,\tau) \frac{C_{00}^{\prime \prime}(u,\tau)}{P^{\prime}(u,T)} \frac{\partial C_{00}(u,\tau)}{\partial u} du d\tau; \\ C_{11}(x,t) &= -\frac{2\pi}{L^{2}} \sum_{n=1}^{\infty} n \ F_{nc} c_{n}(x) e_{nc}(t) \int_{0}^{t} e_{nc}(-\tau) \int_{0}^{t} s_{n}(u) g_{L}(u,T) \frac{\partial C_{01}(u,\tau)}{\partial u} du d\tau; \\ C_{11}(x,t) &= -\frac{2\pi}{L^{2}} \sum_{n=1}^{\infty} n \ F_{nc} c_{n}(x) e_{nc}(t) \int_{0}^{t} e_{nc}(-\tau) \int_{0}^{t} s_{n}(u) g_{L}(u,T) \frac{\partial C_{01}(u,\tau)}{\partial u} du d\tau - \\ -\frac{2\pi}{L^{2}} \sum_{n=1}^{\infty} n \ F_{nc} c_{n}(x) e_{nc}(t) \int_{0}^{t} e_{nc}(-\tau) \int_{0}^{t} s_{n}(u) g_{L}(u,T) \frac{\partial C_{01}(u,\tau)}{\partial u} du d\tau - \\ -\frac{2\pi}{L^{2}} \sum_{n=1}^{\infty} n \ F_{nc} c_{n}(x) e_{nc}(t) \int_{0}^{t} e_{nc}(-\tau) \int_{0}^{t} s_{n}(u) g_{L}(u,T) \frac{\partial C_{01}(u,\tau)}{\partial u} du d\tau - \\ -\frac{2\pi}{L^{2}} \sum_{n=1}^{\infty} n \ F_{nc} c_{n}(x) e_{nc}(t) \int_{0}^{t} e_{nc}(-\tau) \int_{0}^{t} s_{n}(u) g_{L}(u,T) \frac{\partial C_{00}(u,\tau)}{\partial u} du d\tau - \\ -\frac{2\pi}{L^{2}} \sum_{n=1}^{\infty} n \ F_{nc} c_{n}(x) e_{nc}(t) \int_{0}^{t} e_{nc}(-\tau) \int_{0}^{t} s_{n}(u) g_{L}(u,T) \frac{\partial C_{00}(u,\tau)}{\partial u} du d\tau - \\ -\frac{2\pi}{L^{2}} \sum_{n=1}^{\infty} n \ F_{nc} c_{n}(x) e_{nc}(t) \int_{0}^{t} e_{nc}(-\tau) \int_{0}^{t} s_{n}(u) g_{L}(u,T) \frac{\partial C_{00}(u,\tau)}{\partial u} du d\tau - \\ -\frac{2\pi}{L^{2}} \sum_{n=1}^{\infty} n \ F_{nc} c_{n}(x) e_{nc}(t) \int_{0}^{t} e_{nc}(-\tau) \int_{0}^{t} s_{n}(u,T) \frac{\partial C_{00}(u,\tau)}{\partial u} du d\tau - \\ -\frac{2\pi}{L^{2}} \sum_{n=1}^{\infty}$$

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In this section, we calculate the second-order approximations of spatio-temporal distributions of concentrations of dopant and radiation defects. The second-order approximations are usually enough good approximation to make qualitative analysis and to obtain some quantitative results. The result has been confirmed by comparison with the results of numerical simulation and experimental results [12-18,20,21].

# **3. DISCUSSION**

In this section, we analyzed the spatio-temporal distributions of concentrations of dopant and radiation defects in the heterostructure from Figurs 1 and 2 by using the appropriate the second-order approximations from the previous section. We took into account radiation damage during consideration dopant redistribution after ion doping of heterostructure. Figure 3 shows typical distributions concentrations of infused dopant of in heterostructure in a direction perpendicular to the interface between the epitaxial layer and the substrate. The distributions have been calculated under condition, when the value of dopant diffusion coefficient in epitaxial layer was larger than the value of dopant diffusion coefficient in substrate. Figure 4 shows a similar distribution of concentration of dopant corresponding to the ion doping of the heterostructure. Figures 3 and 4 show that the presence of interface between layers of heterostructure under above condition gives us the possibility to obtain thinner field-effect transistor. At the same time, one can find increasing of homogeneity of dopant distribution in doped area.

Where  $\psi(x)$  is the approximation function, which is presented in Figurs 5 and 6 as curve 1. Optimal annealing time was achieved by minimization mean-squared error (12).Dependences of optimal annealing time on parameters related to diffusion of ion types of doping are presented in Figurs 7 and 8. Optimal annealing time which corresponds to ion doping is smaller than the same time for doping by diffusion. The reason of this difference is necessary to anneal radiation defects. Optimization of annealing time for ion doping of materials should be done only in the case when dopant does not achieve interface between layers of heterostructure during annealing of radiation defects.



**Figure 3.** Distributions of concentration of infused dopant in the heterostructure from Figures 1 and 2. Increasing the number of curve corresponds to increasing the difference between values of dopant diffusion coefficient in layers of heterostructure under condition, when the value of dopant diffusion coefficient in epitaxial layer is larger than the value of dopant diffusion coefficient in substrate. Circles are experimental data obtained by Suvar, Christensen, Kuznetsov, and Radamson [31]. Squares are experimental data obtained by Masse and Djessas [32] 2.0



**Figure 4.** Distributions of concentration of implanted dopant in heterostructure from Figures 1 and 2. Curves 1 and 3 correspond to annealing time  $\Theta =$  $0.0048(L_x^2+L_y^2+L_z^2)/D_0$ . Curves 2 and 4 correspond to annealing time  $\Theta = 0.0057(L_x^2+L_y^2+L_z^2)/D_0$ . Curves 1 and 2 correspond to homogenous sample. Curves 3 and 4 correspond to heterostructure under condition, when the value of dopant diffusion coefficient in epitaxial layer is larger than the value of dopant diffusion coefficient in substrate. Circles are experimental data obtained by T. Ahlgren, J. Likonen, J. Slotte, J. Räisänen, M. Rajatore and J. Keinonen [33]. Squares are experimental data obtained by T. Noda [34]

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Figure 5. Spatial distributions of concentration of infused dopant in heterostructure from Figs. 1 and 2. Curve 1 is the idealized distribution of dopant. Curves 2–4 are the real distributions of dopant for different values of annealing time. Increasing the number of curves corresponds to the increasing of annealing time



Figure 7. Dependences of dimensionless optimal annealing time for doping by diffusion, which have been obtained by the minimization of mean-squared error, on several parameters. Curve 1 is the dependence of dimensionless optimal annealing time on a/L for  $\xi = \gamma = 0$  for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 is the dependence ofdimensionless optimal annealing time on value of parameter  $\varepsilon$  for a/L=1/2 and  $\xi = \gamma = 0$ . Curve 3 is the dependence of dimensionless optimal annealing time on value of parameter  $\xi$  for a/L=1/2 and  $\varepsilon = \gamma = 0$ . Curve 4 is the dependence of dimensionless optimal annealing time on the value of parameter  $\gamma$  for a/L=1/2 and  $\varepsilon = \xi = 0$ 



Figure 6. Spatial distributions of concentration of implanted dopant in heterostructure presented in Figs. 1 and 2. Curve 1 is the idealized distribution of dopant. Curves 2–4 are the real distributions of dopant for different values of annealing time. Increasing the number of curves corresponds to the increasing of annealing time



**Figure 8.** Dependences of dimensionless optimal annealing time for doping by ion implantation, which have been obtained by minimization of mean-squared error, on several parameters. Curve 1 is the dependence of dimensionless optimal annealing time on a/L for  $\xi = \gamma = 0$  for equal to each the other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 is the dependence of dimensionless optimal annealing time on the value of parameter  $\varepsilon$  for a/L=1/2 and  $\xi = \gamma = 0$ . Curve 3 is the dependence of dimensionless optimal annealing time on the value of parameter  $\xi$  for a/L=1/2 and  $\varepsilon = \gamma = 0$ . Curve 4 is the dependence of dimensionless optimal annealing time on the value of parameter  $\gamma$  for a/L=1/2 and  $\varepsilon = \xi = 0$ 

$$U = \frac{1}{L} \int_{0}^{L} \left[ C(x, \Theta) - \psi(x) \right] dx, \qquad (12)$$

where  $\psi(x)$  is the approximation function, which presented in Figurs 5 and 6 as curve 1. We determine optimal annealing time by minimization mean-squared error (12). Dependences of optimal annealing time on parameters for diffusion ion types of doping are presented in Figs. 7 and 8. Optimal annealing time, which corresponds to ion doping, is smaller than the same time for doping by diffusion. The reason of this difference is necessity to anneal radiation defects. Optimization of annealing time for ion doping of materials should be done only in this case, when dopant did not achieve interface between layers of heterostructure during annealing of radiation defects.

# 4. CONCLUSION

In this paper, an approach to manufacture thinner field-effect transistors without *p-n*-junctions was considered. Framework for manufacturing included a heterostructure with specific configuration, doping required parts of the heterostructure by dopant diffusion or by ion implantation and optimization of annealing of dopant and/or radiation defects. The optimization gave us the possibility to decrease dimensions of field-effect transistors. An analytical approach was introduced to model technological processes without crosslinking concentrations of dopant and radiation defects on interfaces between layers of heterostructure.

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