Study on Fe$_3$O$_4$ Magnetic Nanoparticles Size Effect on Temperature Distribution of Tumor in Hyperthermia: A Finite Element Method

Shahryar Malekie$^{1,*}$ and Ali Rajabi$^2$

$^1$Radiation Applications Research School, Nuclear Science and Technology Research Institute, PO Box 31485-498, Karaj, Iran.
$^2$Secondary Standard Dosimetry Laboratory (SSDL), Pars Isotope, Karaj, Iran.

(*) Corresponding author: smaleki@aeoi.org.ir
(Received: 05 February 2019 and Accepted: 06 March 2020)

Abstract

In recent years, Hyperthermia has been used as an emerging technique for cancer treatment, especially for localized tumors. One of the promising cancer treatment approaches is magnetic nanoparticle (MNPs) Hyperthermia. In this theoretical work, the temperature distribution of a common tumor over the different sizes of Fe$_3$O$_4$ magnetic nanoparticles, namely 25, 50, 100, and 200 nm, was studied via the finite element method. A two-dimensional method was used to simulate the tumor tissue, in which nanoparticles were incorporated and dispersed into the tumor uniformly. The bio heat transfer equation (BHTE) was applied to calculate the thermal processes in the human body. Results elucidated that decreasing magnetic nanoparticle size caused more temperature rise in the tumor cell during the Hyperthermia treatment, which led to better performance of the treatment. Finally, simulation results showed that the Fe$_3$O$_4$ magnetic nanoparticles with the sizes of 50-100 nm were applicable for Hyperthermia therapy with the optimum cellular uptake.

Keywords: Hyperthermia, Magnetic nanoparticles, Size effect, Tumor, Finite element method.

1. INTRODUCTION

In chemotherapy, generally below the 0.1% of the drugs are absorbed by tumors, while 99.9% delivering into the normal tissue [1]. Hyperthermia therapy refers to body temperatures enhanced to knock out cancer cells. It is always handled with other types of cancer therapy, e.g., chemotherapy and radiation therapy [2]. In Hyperthermia, various techniques attributing to different energies are used, including heat, microwave, laser, ultrasound, and magnetic fields [3]. One of the promising cancer treatment approaches is magnetic nanoparticle (MNPs) Hyperthermia. In fact, by injecting the MNPs such as Fe$_3$O$_4$ in the tumor and subjecting these nanoparticles into a changeable magnetic field, they generate heat, reaching to temperatures up to 42°C that probably cancer cells can be killed, commonly with the lowest damage to the healthy tissue [4]. Consequently, this approach can diminish tumors with negligible side effects [4]. The magnetic fields produced by Hyperthermia studies are limited to 70 mT [5] to 1.5 T [6, 7]. Iron oxide nanoparticles colloids peculiarly magnetite (Fe$_3$O$_4$) and maghemite ($\gamma$-Fe$_2$O$_3$) were studied for magnetic Hyperthermia because of their biocompatibility and high magnetization [8-12]. Several investigations have been carried out on Fe$_3$O$_4$ nanoparticles [13-15]. Mostly, these ferrofluid particles in the form of emulsion injected into a patient [1]. These MNPs have been examined in humans and
animals [16-19]. Several researchers investigated magnetic Hyperthermia [20-30].

The dissipation of the MNPs power in a hysteresis loop is introduced as a specific loss power (SLP) or specific absorption rate (SAR) [31]. Usually, three crucial procedures are taken into account for the heating features of the MNPs [31]. Firstly, Néel relaxation, in which the magnetization of the MNPs is provoked by thermal instabilities [31]. Secondly, Brownian relaxation regarding the rotation of the MNPs in the Brownian motion, and lastly, the external fields [31]. Several factors determine the mechanisms, as mentioned earlier, like the temperature, uniformity, and the frequency of the magnetic field, size, and saturation magnetization of the MNPs [31]. The size of the MNPs is an essential parameter in the magnetic particle Hyperthermia. Particle sizes of Fe₃O₄ nanoparticles can be manipulated by different production procedures ranging from nanometer to micrometer sizes [1, 32-34]. Thapa et al. reported a new method to fabricate the MNPs ranging from 5-100 nm [35]. In the other work, Hilger et al. investigated the breast adenocarcinoma cells embedded into immunodeficient mice via injection of the MNPs into the tumor cells regarding 6.5 kA/m magnetic field at a frequency of 400 kHz, in which exhibited temperature enhances up to 73°C in the tumor domains [36, 37].

In this theoretical work, the evaluation of the temperature distribution of a common tumor is investigated over the different sizes of Fe₃O₄ magnetic nanoparticles with uniform dispersion in the tumor via the finite element method considering the Hyperthermia approach.

1.1. Pennes Equation for Bio Heat Transfer (BHTE)

The equation related to heat transfer in a biomaterial is given by Pennes equation [38]:

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + \sigma E^2 - c_b W_b (T - T_b) + Q_m$$  (1)

Where $\rho$, $c$, $k$, $T$, $c_b$, $W_b$, $T_b$, $Q_m$, and $E$ are density, specific heat capacity, thermal conductivity, temperature, specific heat capacity of blood, blood mass flow rate, the temperature of blood, specific power and electric field respectively [39]; $E$ is obtained via $E = -\nabla V$, where $V$ is the electrical potential which is determined from the solving of the Laplace’s equation ($\nabla^2 V = 0$) in the defined boundary conditions. As can be seen from the Fig. 1, the blood mass flow rate or blood perfusion in the tumor can be calculated as [40]:

$$W_{tumor} = \begin{cases} 
0.8 & \text{T }< 37 \\
0.8 - \frac{(T - 37)^{4.8}}{5400} & 37 \leq T \leq 42 \\
0.38 & \text{T } > 42
\end{cases}$$  (2)

Since the blood mass flow rate in capillaries and tissues depends on their actual temperature, so as can be easily seen from the Fig. 1, it can be deduced that perfusion in tumor tissue in the range of 37-42 °C declines and reaches to its minimum value of 0.38 kg/(s.m³) in 42°C, and after that despite temperature rise, but the value of perfusion remains constant.

![Figure 1. Perfusion rate value in different temperatures for tumor tissue.](image)

Therefore, in Hyperthermia for curing the cancer cells, there is an optimum temperature that there is no need to increase the temperature of tumor tissue necessarily beyond it.
1.2 Magnetic Field and Heating Mechanism

Maxwell’s equations are given by [1]:

\[ \nabla \times \vec{H} = \vec{J} \]  
(3)  
\[ \nabla \cdot \vec{B} = 0 \]  
(4)  
\[ \vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu_0 (\vec{H} + \chi \vec{H}) \]  
(5)

In which \( \vec{B} \), \( \vec{H} \), \( \vec{J} \), \( \vec{M} \), and \( \chi \) are the magnetic field, magnetic intensity, current density, magnetization, and magnetic susceptibility, respectively, while \( \mu_0 \) is the permeability of the vacuum [1]. Due to the negligible amount of \( \chi \) for human body (\( \chi \approx 10^{-6} \) - \( 10^{-4} \)), it is evident that the magnetic field can transmit in the body unchangeably [1]. While Fe\(_3\)O\(_4\) nanoparticles exhibit higher magnetic susceptibilities (\( \chi \approx 20 \)) [1]. The force applied to a magnetic nanoparticle is introduced as [1]:

\[
\vec{F}_m = \frac{4\pi a^3}{3} \frac{\mu_0 \chi}{1 + \chi/3} \left[ \frac{d\vec{H}}{dx} \right] \vec{H} 
\]

(6)

which \( a \) is the radius of a nanoparticle.

As can be seen from Fig. 2, the alignment of MNPs is affected by the applied magnetic field. The spatial orientation of the MNP is described by the respective polar and azimuthal angles of \( \theta \), \( \phi \), and magnetization relative to the applied field [41]. Usually, in Hyperthermia, the magnetic field is ranging from 70 mT [5] to 2.2 T [1, 42].

Diffusion coefficient of a tissue (\( D_B \)) exhibits an inverse relationship with the particle radius [1]:

\[ D_B = \frac{k_B T}{6\pi \eta a} \]  
(7)

which \( k_B \), \( T \), \( a \), and \( \eta \) are the Boltzmann constant, temperature, particle radius, and fluid viscosity [1].

Figure 2. The schematic view of a magnetic nanoparticle influenced by the magnetic force on its magnetization orientation.

The value of produced heat per unit volume in a hysteresis process is given by [16]:

\[ P = \mu_0 f \int \vec{H} \, d\vec{M} \]  
(8)

which, \( f \) is the frequency of the alternating magnetic field ranging from \( f=0.05-1.2 \) MHz and \( H=0-15 \) kA m\(^{-1}\) conventionally [16].

2. SIMULATION METHODOLOGY

2.1 Uniform Distribution of MNPs

This simulation considers dispersed MNPs of Fe\(_3\)O\(_4\) with sizes of 25, 50, 100, and 200 nm in typical tumor tissue with an elliptical cross-section according to Fig. 3. For this purpose, we considered a 2D array of nanoparticles with a size of 4 \( \mu \)m\( \times \)4 \( \mu \)m in which the number of nanoparticles was exhibited in Table 1 with a constant volume fraction of 7.4% in the tumor cell. The dispersion state of the MNPs of Fe\(_3\)O\(_4\) in the tumor is such that these particles satisfy the excluded volume approach in order to prevent from the overlapping or penetration of nanoparticles to each other. In this simulation, an AC magnetic field was imported into the problem via the Pennes equation (Eq.1). It can be assumed that the specific power (\( Q_m \)) in the Eq.1 and the value of the produced heat per unit volume (\( P \)) in the Eq.8 are equivalent. Also \( P \) and \( Q_m \) have the same unit of W/m\(^3\). Measurement of the Neel relaxation of magnetic particles in the frequency ranging from 1 kHz to 160 MHz was introduced by Fannin and et al. [43]. So, this simulation
was implemented for the case that \( f=100 \text{ kHz} \), \( H=4 \text{ kA/m} \), and \( \chi = 20 \). It can be calculated that from the Eq.5, \( B=105 \text{ mT} \), that is within the range of the other reported investigation [5]. In this simulation, \( H \cdot f = 4.0 \times 10^8 \text{ Am}^{-1}\text{s}^{-1} \) that satisfies the ‘Atkinson-Brezovich criterion’ in which \( H \cdot f = 4.85 \times 10^8 \text{ Am}^{-1}\text{s}^{-1} \). This issue ensures the safe and tolerable amount of magnetic field applied for humans [4, 44]. The physical quantities have been exhibited in Table 2.

![Figure 3. Schematic view of the geometry in this model.](image)

**Table 1. The number of MNPs and their sizes in a square lattice.**

<table>
<thead>
<tr>
<th>Array</th>
<th>Number of nanoparticles</th>
<th>( r ) (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5\times{}5</td>
<td>25</td>
<td>200</td>
</tr>
<tr>
<td>10\times{}10</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>20\times{}20</td>
<td>400</td>
<td>50</td>
</tr>
<tr>
<td>40\times{}40</td>
<td>1600</td>
<td>25</td>
</tr>
</tbody>
</table>

**Table 2. Physical quantities and their values in this simulation[45].**

<table>
<thead>
<tr>
<th>Layer</th>
<th>Thermal Conductivity (W/m.K)</th>
<th>Heat Capacity (J/kg.K)</th>
<th>Density (kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Healthy Tissue</td>
<td>0.51</td>
<td>3760</td>
<td>1045</td>
</tr>
<tr>
<td>Tumor</td>
<td>0.51</td>
<td>3760</td>
<td>1045</td>
</tr>
<tr>
<td>( \text{Fe}_3\text{O}_4 )</td>
<td>2000.6</td>
<td>644</td>
<td>5</td>
</tr>
</tbody>
</table>

**2.2. Finite Element Method**

The finite element method (FEM) is a robust numerical technique for solving boundary-value problems (BVPs) [46]. In FEM, the domain is divided into small subdomains or elements, which are related to each other by the nodes [47]. During Hyperthermia therapy, finding the precise answer of the BHTE for a tumor tissue is a challenging issue, so FEM can be useful to obtain the temperature distribution in the tumor tissue. A schematic exhibition of 2D mesh processing of this simulation is depicted in Fig. 4. In order to find the electrical potential of the elements, the Laplace equation was solved. In this simulation, a personal computer having 32 GB RAM and 3.4 GHz processor was used to predict the temperature distribution in tumor tissue numerically.

![Figure 4. Mesh quality simulated in this research.](image)

**3. RESULTS AND DISCUSSION**

Fig. 5 plots the thermal distribution in healthy tissue and tumor for the \( \text{Fe}_3\text{O}_4 \) MNPs with different sizes. Considering Fig. 5, the amount of normal tissue temperature remains constant at 37°C, while the temperature of the tumor domain increases with the decrease of the MNP size.

According to Table 3, it can be concluded that decreasing MNP size resulted in a temperature rise in the tumor cell during the Hyperthermia treatment. As can be easily seen from Table 3, the maximum temperature in the tumor cell is
related to the minimum nanoparticle size of 25 nm. To justification of this phenomenon, it can be mentioned those small size particles due to 25 nm exhibit a more massive amount of surface to volume ratio; thus, the significant discrepancy between thermal conductivities of tissue and the Fe$_3$O$_4$ MNPs results in increasing the temperature in the tumor tissue.

![Figure 5. Thermal distribution of Fe3O4 MNPs with different sizes in healthy tissue and tumor.](image)

Also, ‘Brownian rotation’ of the MNPs causes to increase the temperature in the tumor region [16].

Maybe the higher temperature rise due to smaller particle sizes is related to a more considerable amount of ‘Brownian rotation’ for small particles or lower amount of rotational inertia of the MNP, regarding the fact that rotational inertia of a body is equal to the product of mass and square of the distance of the object to the rotation axis. The moment of inertia for nanoparticles can be calculated using the standard definition in mechanics [48]. It can be mentioned that small particles with sizes up to 25 nm generally present superparamagnetic mode that stimulates a reduction in the agglomeration when the magnetic fields are eliminated [1].

It should be considered a compromise between the size of the MNPs and the maximum temperature rise in the tumor.

Several related experiments [49, 50], and simulations [51-54], about the magnetic nanoparticles with different sizes for Hyperthermia have been reported.

As can be easily seen from Fig. 1, there is an optimum temperature that there is no need to increase the temperature of tumor tissue necessarily beyond it, so decreasing the MNPs size or increasing the maximum temperature in tumor cells subsequently is not adequate.

For larger nanoparticles more than 100 nm, the cellular uptake is decreased because of the larger nanoparticles unable to diffuse into the tumor cells quickly. Regarding this fact, it can be mentioned that the MNPs with the sizes of 50-100 nm are practical for Hyperthermia therapy.

<table>
<thead>
<tr>
<th>Magnetic nanoparticle radius (nm)</th>
<th>Maximum temperature in the tumor (°C)</th>
<th>Temperature rise $\Delta T$ (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>61</td>
<td>24</td>
</tr>
<tr>
<td>50</td>
<td>49</td>
<td>12</td>
</tr>
<tr>
<td>100</td>
<td>43</td>
<td>6</td>
</tr>
<tr>
<td>200</td>
<td>41</td>
<td>4</td>
</tr>
</tbody>
</table>

$^1$ The thermal difference between normal (37°C) and tumor tissues

4. CONCLUSION

In summary, the temperature distribution of a common tumor over the different sizes of Fe$_3$O$_4$ MNPs, namely 25, 50, 100, and 200 nm were studied regarding the finite element procedure. A two-dimensional method was used to simulate the tumor tissue, in which nanoparticles were incorporated and dispersed into the tumor uniformly. The bio heat transfer equation was applied to calculate the thermal processes in the human body. Results showed that decreasing of magnetic nanoparticle size caused to temperature rise in the tumor cell during the Hyperthermia treatment or better performance of the treatment. One of the benefits of this method is that the temperature of the healthy tissue remains constant at 37°C, and only the temperature of the tumor domain is increased by
incorporating the magnetic nanoparticles into the tumor. Regarding this fact, it can be mentioned that the magnetic nanoparticles of Fe3O4 with the sizes of 50-100 nm are practical for Hyperthermia therapy with the optimum cellular uptake.

Conflicts of Interest
The authors declare that there are no conflicts of interest regarding this article.

REFERENCES