Optimization of Thermal Conductivity of $\text{Al}_2\text{O}_3$ Nanofluid by Using ANN and GRG Methods

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Abstract:
Common heat transfer fluids such as water, ethylene glycol, and engine oil have limited heat transfer capabilities due to their low heat transfer properties. Nanofluids are suspensions of nanoparticles in base fluids, a new challenge for thermal sciences provided by nanotechnology. In this study, we are to optimize and report the effects of various parameters such as the ratio of the thermal conductivity of nanoparticles to that of a base fluid, volume fraction, nanoparticle size, and temperature on the effective thermal conductivity of nanofluids using nonlinear optimization methods and artificial neural network. The results for nonlinear optimization methods show that Thermal conductivity of nanofluid enhanced by 32 percent. For the modeling of the Thermal conductivity of nanofluid, the feed-forward back-propagation ANN is employed. Result showed the maximum enhancement of 42 percent for thermal conductivity and this method is more acceptable since excellent agreement between the predictions and the experimental data is obtained with a MAE (mean absolute error) of 0.30%.

Keywords: $\text{Al}_2\text{O}_3$ nanofluid, Thermal conductivity, Nonlinear optimization, Neural networks.

1. INTRODUCTION

Common heat transfer fluids such as water, ethylene glycol, and engine oil have limited heat transfer capabilities due to their low heat transfer properties. Numerous researchers have been investigating better ways to enhance the thermal performance of heat transfer fluids. One of the methods used is to add nano-sized particles of highly thermally conductive materials like carbon, metal, metal oxides into the heat transfer fluid to improve the overall thermal performance of the fluid. Nanofluids are created by suspending nanometer-sized particles (less than 100 nm) in a pure fluid such as water, ethylene glycol, or propylene glycol. The effect is observed when nanoparticles have heat conductivity many times greater than the liquid. Usually Cu, Ag, CuO, $\text{Al}_2\text{O}_3$, or CNT are used. The advantages of using nanoparticles are that they are more easily suspended in the fluid, they may be used in microchannels, and the small size causes less wear to machinery. However, aggregation of particles must be minimized in order to benefit from these effects of small particle size.

Zhu et al. [1] demonstrated that the stability and thermal conductivity of $\text{Al}_2\text{O}_3$/water nanofluids are highly dependent on pH values and the different SDBS dispersant concentrations of nanosuspensions. Several reports for the thermal conductivity of nanofluid is shown in Table 1.


Table 1. Several reports for the thermal conductivity

<table>
<thead>
<tr>
<th>Researcher</th>
<th>Base fluid</th>
<th>Nanoparticle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Masuda et al. [2]</td>
<td>Distilled water</td>
<td>Al₂O₃, TiO₂</td>
</tr>
<tr>
<td>Xie et al. [3]</td>
<td>Distilled water</td>
<td>SiC</td>
</tr>
<tr>
<td>Das et al. [4]</td>
<td>Distilled water</td>
<td>Al₂O₃, CuO</td>
</tr>
<tr>
<td>Murshed et al. [5]</td>
<td>Distilled water</td>
<td>TiO₂</td>
</tr>
<tr>
<td>Wen and Ding [6]</td>
<td>Distilled water</td>
<td>Al₂O₃</td>
</tr>
<tr>
<td>Patel et al. [7]</td>
<td>Distilled water</td>
<td>Ag, Au</td>
</tr>
<tr>
<td>Jamal-Abadi [8]</td>
<td>Distilled water</td>
<td>Al, Cu</td>
</tr>
</tbody>
</table>

Effective thermal conductivity of blood with suspension of Al₂O₃ nanoparticles as a bio-nanofluid was studied by Ghassemia et al [9]. They were shown that the results of the proposed model for the blood thermal conductivity agree well with the available data in the literature.

Experimental investigations and theoretical determination of effective thermal conductivity and viscosity of Al₂O₃/H₂O nanofluid are reported by Chandrasekara et al [10]. Al₂O₃/water nanofluid with a nominal diameter of 43 nm at different volume concentrations (0.33–5%) at room temperature were used for the investigation. Results were shown that both the thermal conductivity and viscosity of nanofluids increase with the nanoparticle volume concentration, meanwhile the viscosity increase is substantially higher than the increase in thermal conductivity.

Thermal conductivity and viscosity of the Al₂O₃/R141b nanorefrigerant for 0.5 to 2 vol.% concentrations at temperatures of 5 to 20°C have been investigated by Mahbubula et al [11]. The experimental results show that, thermal conductivity of the Al₂O₃/R141b nanorefrigerant increased with the augmentation of particle concentrations and temperatures. Thermal conductivity of ethylene glycol and water mixture based Fe₃O₄ nanofluid has been investigated experimentally by Sundara et al [12]. Nanofluids were prepared by dispersing nanoparticles into different base fluids like 20:80%, 40:60% and 60:40% by weight of the ethylene glycol and water mixture. Results indicate that the thermal conductivity increases with the increase of particle concentration and temperature. Furthermore, the theoretical Hamilton–Crosser model failed to predict the thermal conductivity of the nanofluid with the effect of temperature. the effect of particle size, temperature, and weight fraction on the thermal conductivity ratio of alumina(Al₂O₃)/water nanofluids were studied by Tenga et al [13]. This experiment measured the thermal conductivity of nanofluids with different particle sizes, weight fractions, and working temperatures (10, 30, 50°C). The results showed a correlation between high thermal conductivity ratios and enhanced sensitivity, and small nanoparticle size and higher temperature. In this paper, the effect some parameter on the thermal conductivity ratio of alumina nanofluids were investigated and optimum of thermal conductivity ratio was obtained. using nonlinear optimization methods and artificial neural network for approaching to this aim.

2. VARIABLES OPTIMIZED

2.1. Volume fraction

The thermal conductivity of Al₂O₃ nanofluids increases with an increase of volume fraction. For example, according to figure 1, for the 3 vol% Al₂O₃ nanofluid, the relative thermal conductivity increases approximately 6% compared to the base fluid [14].

![Figure 1: Effect of particle volume concentration for two group of Al₂O₃ nanofluid](image-url)
2.2. Particle size

The effect of particle size on thermal conductivity enhancement of Al₂O₃ particle/water combination over a range of particle diameter from 28 to 60 nm is shown in figure 2. Graph demonstrates that larger thermal conductivity enhancement is produced by bigger particles. However, the results for the 28 nm particle fall between the two larger sizes [14, 4].

![Figure 2: Effect of particle size for Al₂O₃ in water.](image)

2.3. Temperature

The influence of temperature on the enhancement in thermal conductivity of nanofluid is shown in figure 3. The increase in the effective thermal conductivity of nanofluid with temperature is clearly seen. In this result particle size is an important parameter on thermal conductivity because the opposite temperature trend was reported by Masuda et al. for smaller particles [2]. Also results suggest that the relative increase in thermal conductivity is more important at higher temperature as well as smaller diameter particles. This trend is encouraging for engineering application where fluids operate at elevated temperature.

![Figure 3: Effect of temperature for Al₂O₃ in water.](image)

3. OPTIMIZATION

3.1. General Reduced Gradient Method (GRG)

The Generalized Reduced Gradient method [15] has been developed and tested to be one of the efficient and effective techniques for the Non-linear Programming problem with Non-linear constraints. With the superior properties, this idea is interesting for other researchers.

The idea of generalized reduced gradient is to convert the constrained problem into an unconstrained one by using direct substitution. If direct substitution is possible it will reduce the number of independent variables and remove the constraint equations. There, the constraint equations are expanded in a Taylor series, and only the first order terms are retained. Then with these linear equations, the constraint equations can be used to reduce the number of independent variables.

Lasdon used the idea of binding constraints to refine the GRG method and included the concept of Quasi-Newton method for the search direction [16]. Sandgren compared twenty four procedures that include four GRG algorithms for his suggested testing problems [17]. Haggag reported the application of GRG method in the real-life problems [18]. Other researchers compared GRG technique with other methods.

3.2. The GRG Algorithm:

1. Divide the variables into independent Xᵢ and dependent Xₒ variables and set initial guess as Xₒ=(Xₒ₀, Xₒ₀)

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2. Determine search directions
   a) calculate the reduced gradient
   \[
   g_R = \left[ \frac{\partial f(x)}{\partial x_1} \right] - \left[ \frac{\partial f(x)}{\partial x_D} \right] \left[ \left[ \frac{\partial h(x)}{\partial x_D} \right]^{-1} \right]^T \left[ \frac{\partial f(x)}{\partial x_D} \right] \tag{1}
   \]
   b) determine the search direction for \(X_i\)
      \(\text{(lower limit)}\), \(X_i<\text{(upper limit)}\)
   \[
   s_I^k = -g_I^k \tag{2}
   \]
   c) determine the search direction for \(X_D\)
   \[
   s_D^k = -\left[ \frac{\partial h(x)}{\partial x_D} \right]^{-1} \left[ \frac{\partial h(x)}{\partial x_1} \right] s_I^k \tag{4}
   \]
3. Calculate \(X_{k+1}\)
   \[
   X_i^{k+1} = X_i^k + \lambda^k s_I^k (5)X_D^{k+1} = X_D^k + \lambda^k s_D^k \tag{5}
   \]
\(\lambda\) is determined by using the necessary condition
4. Regain feasibility of dependent variable
   \[
   h(X_D^{k+1}, X_D^{k+1}) \approx h(X_D^{k+1}, X_D^{k+1}) + \frac{\partial h(X_D^{k+1}, X_D^{k+1})}{\partial X_D} (X_D^{k+1} - X_D^{k+1}) \]
   \[
   \Rightarrow \tilde{X}_D^{k+1} = X_D^{k+1} - \left[ \frac{\partial h(X_D^{k+1}, X_D^{k+1})}{\partial X_D} \right]^{-1} h(X_D^{k+1}, X_D^{k+1}) \tag{6}
   \]
5. The procedure terminates if the convergence criterion is satisfied, otherwise goes to step 2.
   In this regard, a computer program in the MATLAB language is written for the GRG algorithm.

### 3.3. Neural networks

Neural networks, with their remarkable ability to derive meaning from complicated or imprecise data, can be used to extract patterns and detect trends that are too complex to be noticed by either humans or other computer techniques. Artificial neural networks (ANNs) have been used in many engineering applications because of providing better and more reasonable solutions [19-22].

Dietz et al. carried out early applications of neural networks to aircraft engine diagnostics [23]. Also, neural networks were used in space main engine by Whitehead et al. [24,25]. Different neural networks have been used in gas turbine engine fault detection, diagnosis and accommodation since then [26-28]. Analysis of thermosiphon solar water heaters, prediction of wall superheat in a reboiler tube and heat transfer data analysis are other sample. Thus it is understood from the literature that ANNs better serve to thermal analysis in engineering applications.

The most basic and commonly used ANN is the multi-layer perceptron (MLP). This consists of at least three or more layers, an input layer, an output layer, and a number of hidden layers. In this regard, for the modeling of the Thermal conductivity of nanofluid use, the feed-forward back-propagation ANN is employed. The back-propagation algorithm of the ANN modeling is considered the most suitable method for training multilayer feed-forward networks.

**Figure 4:** Schematic diagram of multilayer feed forward neural network.

Feed-forward ANNs tend to be straightforward networks that associate inputs with outputs. Feed-
In this research, we worked with the toolbox of sigmoid neurons followed by an output layer of linear neurons. The multilayer feedforward network as shown in figure 4 had three input neurons which corresponded to volume fraction, particles size and temperature and the output layer had one neuron which was thermal conductivity.

In this study, we worked with the toolbox for neural network of the Matlab software [29], using the Levenberg–Marquardt algorithm, considered by Hagan and Menhaj as the most efficient [30,31]. Both the input and output data sets were normalized by Mean & Standard Deviation (M&SD) techniques because it has significant effects on learning accuracy and on convergence [32]. As in nature, the network function is determined largely by the connections between elements. Neural network can be trained to perform a particular function by adjusting the values of the connections (weights) between elements. Each input is weighted with an appropriate \( w \). The sum of the weighted inputs and the bias form the input for the transfer-function and linear transfer function were applied for the hidden layer and the output layer, respectively [33]. Sigmoid function \( f(z) \), where \( Z \) is the weighted sum of the inputs. Thus:

\[
f(z_i) = \frac{1}{1 + e^{-z_i}}
\]

\[
z_i = \sum_{j=1}^{n} w_{ij} x_j + \beta_i
\]

Where \( x_j \) is the incoming signal from the \( j \)th neuron (at the input layer), \( w_{ij} \) the weight on the connection directed from neuron \( j \) to neuron \( i \) (at the hidden layer) and \( \beta_i \) the bias of neuron \( i \). When testing the pertinence of our model, experimental database was split into learning database (70% of experimental data set) and testing database (30% of experimental data set) to get a good representation of the diversity situation. Predictions of the networks performances were evaluated using mean square error (MSE), mean absolute error (MAE), sum of the squares error (SSE) and statistical coefficient of multiple determination (R²) values, which were calculated by the following expressions.

\[
MAE = \frac{1}{n} \sum_{j=1}^{n} |t_j - o_j| = \frac{1}{n} \sum_{j=1}^{n} |e_j|
\]

Where \( e_j \) the mean absolute error

\[
R^2 = 1 - \left( \frac{\sum_j (t_j-o_j)^2}{\sum_j (o_j)^2} \right)
\]

\[
SSE = \sum_j (o_j - t_j)^2
\]

\[
MSE = \frac{1}{n} \sum_{j=1}^{n} (o_j - t_j)^2
\]

Where \( t \) is the target (network output) value, \( o \) is the output (desired) value.

4. RESULTS AND DISCUSSION

In this study we are to optimize and report the effects of various parameters such as volume fraction, nanoparticle size, and temperature on the effective thermal conductivity of nanofluids using two optimization methods. Alumina/water nanofluid data presented by Yu et al. was used as input to conduct this research [34]. Two different methods used in this research are generalized reduced gradient method and artificial neural network. Constrain functions obtained by data surface fitting, are shown in Table 2. The second column of the Table represents the correlation coefficient (R-value) for each function.

<table>
<thead>
<tr>
<th>Constrain</th>
<th>R-square</th>
</tr>
</thead>
<tbody>
<tr>
<td>K((\varphi),s)</td>
<td>0.9991</td>
</tr>
<tr>
<td>K((\varphi),T)</td>
<td>0.8293</td>
</tr>
<tr>
<td>K((\varphi),pH)</td>
<td>0.9915</td>
</tr>
</tbody>
</table>

Results attained using the first method showed that output is very sensitive to initial values and by changing the initial guess, optimized values changed dramatically. Thermal conductivity of nanofluid enhanced by 32 percent at \( \varphi=6.86 \) vol\%, \( T=47.70 \) and \( s=10 \) nm. These results are not reliable since this method finds local optimum values and does not search throughout the field for optimum values.
The predictions of trained ANN for temperature, concentration and size; the solutions as a function of the experimental ones are demonstrated in figure 5 for the training data set. To assess the accuracy of the ANN predictions, each graph is provided with a straight line indicating the perfect prediction. The performance of the neural network prediction was evaluated by a regression analysis between the predicted and the experimental values. The ANN predictions produce the statistical coefficient of multiple determinations (R²) in the range of 0.99 and mean absolute error (MAE) in the range of 0.00018. For the training data set, as shown in the Table 3.

The R² and MAE values are within an acceptable range. A comparison of the ANN predicted and experimentally measured thermal conductivity values for testing data set are shown in figure 6. The ANN prediction and experimental values yield R² of 0.94 and MAE of 0.0031 as shown in Table 2.

The regression coefficients obtained from testing of the ANN were perfect and within the acceptable limits in both cases. As the correlation coefficient approaches to 1, the accuracy of the prediction improves. In the presented case, the correlation coefficients range is very close to 1. There was excellent agreement between the predicted values and the measured values; which indicates excellent agreement between the experimental and the ANN predicted results. The low value of the average mean absolute error also shows the accuracy of the result. This result indicates that the performance of the network output is excellent.

Totally different results obtained using genetic algorithm method. It showed the maximum enhancement of 42 percent for thermal conductivity at φ = 4.5 vol%, T = 48°C and s = 36 nm. Although these results are more trustworthy due to superiority of artificial neural network, we need more experimental data to ensure about their precision. Table 4 shows comparison between results of artificial neural networks method and generalized reduced gradient method, volume fraction and temperature for both method are approximately equal, but there is a noticeable difference in nanoparticle size. The optimal of thermal conductivity enhancement is predicted 42% and 32% by ANN and GRG method respectively.

![Figure 5: Comparison of experimental data and ANN-predicted values of thermal conductivity for the training data set](image1)

![Figure 6: Comparison of experimental data and ANN-predicted values of thermal conductivity for the test data set](image2)

**Table 3: Performance of neural network for prediction of thermal conductivity**

<table>
<thead>
<tr>
<th></th>
<th>training</th>
<th>test</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAE</td>
<td>0.0031</td>
<td>0.00018347</td>
</tr>
<tr>
<td>MSE</td>
<td>0.000014701</td>
<td>0.00000024256</td>
</tr>
<tr>
<td>SSE</td>
<td>0.00022051</td>
<td>0.00021587</td>
</tr>
<tr>
<td>R²</td>
<td>0.9169</td>
<td>0.9982</td>
</tr>
</tbody>
</table>
Table 4: Comparison between results of artificial neural network and generalized reduced gradient method.

<table>
<thead>
<tr>
<th>Method</th>
<th>ANN</th>
<th>GRG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume fraction (%)</td>
<td>4.5</td>
<td>6.86</td>
</tr>
<tr>
<td>Temperature (°C)</td>
<td>48</td>
<td>47.7</td>
</tr>
<tr>
<td>Size (nm)</td>
<td>36</td>
<td>10</td>
</tr>
<tr>
<td>Thermal conductivity enhancement (%)</td>
<td>42</td>
<td>32</td>
</tr>
</tbody>
</table>

5. CONCLUSION

This paper presents an application of the GRG method and artificial neural networks in order to optimize the thermal conductivity of Al₂O₃ nanofluid solutions based on the temperature, concentration and size. In this study, we are to prove whether GRG and ANNs method can be used for the Nonlinear Optimization of thermal conductivity of the nanofluid or not. The principal conclusions are as follows:

1. 32% increase for thermal conductivity of Al₂O₃ nanofluid were observed by GRG method and 42% by ANN method.
2. ANN compare with GRG is more accuracy for the Optimization of thermal conductivity of the Al₂O₃ nanofluid–water.
3. because of their nature, artificial neural networks provide better expectations to detect global optimum solutions than gradient methods.
4. The performances of the ANN prediction and experimental results were measured using the mean absolute error (MAPE), sum of the squares error (SSE), the statistical coefficient of multiple determination or correlation coefficients (R²). A good regression analysis with the R² in the range of 0.99 and MAE in the range of 0.00018 and 0.003 for the test data set and training was obtained, respectively.
5. This study helps application engineers determine the thermal conductivity of the Al₂O₃ nanofluid easily without exhaustive experiments, thus saving both money and time.

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