Numerical Simulation of MHD Boundary Layer Stagnation Flow of Nanofluid over a Stretching Sheet with Slip and Convective Boundary Conditions

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Abstract
An investigation is carried out on MHD stagnation point flow of water-based nanofluids in which the heat and mass transfer includes the effects of slip and convective boundary conditions. Employing the similarity variables, the governing partial differential equations including continuity, momentum, energy, and concentration have been reduced to ordinary ones and solved by using Keller-Box method. The behavior of emerging parameters is presented graphically and discussed for velocity, temperature, and nanoparticles fraction. The numerical results indicate that for the stretching sheet, the velocity at a point decreases with the increase in the values of λ and M; whereas both temperature and nanoparticle concentration increase with the increase in velocity slip parameter (λ), magnetic parameter (M) and convective parameter (γ). And also, observed that the velocity profile increases with the increase in velocity ratio parameter.
Keywords: *MHD, Stretching sheet, Nanofluid, Velocity slip, Convective boundary condition.*

NOMENCLATURE:

u, v	: Velocity components in the x- and y-axis, respectively (m/s)	T_f	: Thermal conductivity of the fluid
U_w	: Velocity of the wall along the x-axis (m/s)	h_f	: Convective heat transfer coefficient
x, y	: Cartesian coordinates measured along the stretching sheet. (m)	T_∞	: Temperature of the fluid far away from the stretching sheet (K)
$B(x)$	: Magnetic field strength (A m^{-1})	Re_x	: Reynolds number
C	: Nanoparticle concentration (mol m^{-3})	**Greek Symbols:**	
C_{fx}	: Skin-friction coefficient (pascal)	α	: Thermal diffusivity (m^2/s)
Nu_x	: Nusselt number	σ	: Electrical conductivity (S m^{-1})
Sh_x	: Sherwood number	σ^*	: Stefan-Boltzmann constant
C_w	: Nanoparticles concentration at the	ψ	: Stream function
		η	: Dimensionless similarity variable

stretching surface (mol m^{-3})

C_∞ : Nanoparticle concentration far from the sheet (mol m^{-3})

C_p : Specific heat capacity at constant pressure (J Kg^{-1} K)

D_T : Brownian diffusion coefficient

D_B : Thermophoresis diffusion coefficient

Ec : Eckert number

a : Constant parameter

f : Dimensionless stream function

k^* : Rosseland mean absorption coefficient

k : Thermal conductivity (W m^{-1} K^{-1})

Le : Lewis number

M : Magnetic parameter

Nb : Brownian motion parameter

Nt : Thermophoresis parameter

Pr : Prandtl number

R : Radiation parameter

N_1 : Velocity slip factor

A : Velocity ratio parameter

T : Fluid temperature (K)

q_w : Surface heat flux (W/m^2)

q_m : Surface mass flux

γ : convective parameter

μ : Dynamic viscosity of the base fluid (kg/m.s)

υ : Kinematic viscosity (m^2 s^{-1})

ρ_f : Density of the fluid (Kg m^{-3})

ρ_p : Density of the nanoparticle (Kg m^{-3})

τ : The ratio of the nanoparticle heat capacity & the base fluid heat capacity

$(\rho c)_f$: Heat capacity of the base fluid (kg/m.s^2)

$(\rho c)_p$: Heat capacity of the nanoparticle (kg/m.s^2)

θ : Dimensionless temperature (K)

p : pressure (N/ m^2)

ϕ : Nanoparticle volume fraction

ϕ_W : Nanoparticle volume fraction at wall temperature

ϕ_∞ : Ambient nanoparticle volume fraction

λ : Velocity slip parameter

Subscripts:

f : Fluid

W : Condition on the sheet

∞ : Ambient Conditions

1. INRODUCTION

The problem of flow and heat transfer of an incompressible viscous fluid in the boundary layer flow over a stretching surface is significant in industries and manufacturing process such as the extrusion of polymers, the aerodynamic extrusion of plastic sheets, floating or spinning of fibers are processed, the cooling of metallic plates etc. Sakiadis [1] initiated the study of the boundary layer flow over a continuous solid surface moving with constant speed. Crane [2] is the first present of a self-similar solution in the closed analytical form for steady two-dimensional incompressible boundary layer flow caused by the stretching plate whose velocity varies linearly with the distance from a fixed point on the sheet. Gupta and Gupta [3] investigated the heat and mass transfer on a stretching sheet with suction or blowing. Cortell [4] investigated on fluid flow and radiative non-linear heat transfer over a stretching sheet. Subhas and Veena [5] studied on visco-elastic fluid flow and heat transfer in a porous medium over a stretching sheet.

In that work, he analyzed two cases are the sheet with prescribed surface temperature (PST-Case) and the wall heat flux (PHF-Case).

Nanofluid represents the fluid in which nano-scale particles are engineered colloidal suspensions in a base fluid with low thermal conductivity such as water, ethylene glycol, oil etc. The use of additive is a technique applied to enhance the heat transfer performance of base fluids. Nanofluids have been shown to increase the thermal conductivity and convective heat transfer performance of base liquids. There are several biomedical applications that involve nanofluids such as magnetic cell separation, drug delivery, hyperthermia and contrast enhancement in magnetic resonance imaging. The nanofluid term was coined by Choi [6]. Sheikholeslami et al. [7] investigation of heat transfer on water nanofluid flows in a semi annulus enclosure using Lattice Boltzmann method. Hamad and Fedrow [8] investigated the similarity solution of heat transfer and viscous flow of nanofluid over a non-linearly stretching sheet using RK method. Rana and Bhargava [9] conducted similar research for a nonlinear stretching sheet using finite element and finite difference methods. Makinde and Aziz [10] investigated the boundary layer flow of a nanofluid past a stretching sheet with convective boundary conditions with the help of RK method. Rashidi et al. [11] analyzed the buoyancy effect on MHD flow over a stretching sheet of a nanofluid in the effect of thermal radiation. Sheikholeslami and Rokni [12] studied effect of Lorentz forces on nanofluid flow in a porous complex shaped enclosure by means of non-equilibrium model using control volume based finite element method (CVFEM).

Stagnation point flow has various practical applications. These applications include the cooling of electronic devices, the cooling of nuclear reactors during the emergency shutdown, hydrodynamic processes in engineering applications. Again the study of magnetohydrodynamic (MHD) flow of an electrically conducting fluid is of considerable interest in metallurgical and metalworking processes due to the fact that the rate of cooling can be controlled by the application of magnetic field. Hydromagnetic stagnation point flow and heat transfer finds applications in boundary layers along material handling conveyors, in aerodyne-mic extrusion of plastic sheets and in blood flow problems. Hiemenz [13] considered a two-dimensional stagnation flow problem on a stationary plate and used similarity transformations to reduce the Navier-Stokes equations to nonlinear ordinary differential equations. Akbar et.al [14] investigated on radiation effects on MHD stagnation point flow of nanofluid towards a stretching surface with convective boundary condition. Batti et al.[15] studied a robust numerical method for solving stagnation point flow over a permeable shrinking sheet under the influence of MHD.

Bhattacharyya and Layek [16], Ibrahim et al. [17], Bachok et al. [18] studied the boundary layer problems of stagnation point flow over a stretching or shrinking sheet. Sheikholeslami and Rokni [19] analyzed magnetic nanofluid flow and convective heat transfer in a porous cavity considering brownian motion effects. Sachin Shaw [20] explained Effects of slip on nonlinear convection in nanofluid flow on stretching surfaces. Samir Kumar and Mahapatra [21] studied the effects of slip and heat generation/absorption on MHD stagnation flow of nanofluid past a stretching/shrinking surface with convective boundary conditions. Stagnation electrical MHD nanofluid mixed convection with slip boundary on a stretching sheet was studied by Kai-Long Hsiao [22]. Mustafa et al. [23] studied slip effects of nanofluid in a channel with wall properties on the Peristaltic motion by using homotopy analysis method. Recently velocity slip and thermal jump effects on unsteady stagnation point flow of a nanofluid over a stretching sheet are investigated numerically by Malvandi et al. [24]. Heat transfer improvement and pressure drop during condensation of refrigerant-based nanofluid was studied experimentally by Sheikholeslami et al. [25]. Sheikholeslami and Ghasemi [26] derived on Solidification heat transfer of nanofluid in existence of thermal radiation using finite element method. Several other studies have addressed various aspects of nanofluids (including comparison) with stretching sheet ([27]-[35]). Gangaiah et al. [36] described the effects of thermal radiation and heat source/sink parameters on the mixed convective MHD flow of a Casson nanofluid with zero normal flux of nanoparticles over an exponentially stretching sheet along with convective boundary condition. Ghozatloo et al. [37] studied chemical vapour deposition (CVD) method in atmosphere pressure (14.7 psi) using synthesis of Graphene. Ramya Dodda et al. [38] studied the steady two-dimensional flow of a viscous nanofluid of magnetohydrodynamic (MHD) flow and heat transfer characteristics for the boundary layer flow over a nonlinear stretching sheet is considered. Sheikhol-eslami et al. [39] applied Homotopy perturbation method to investigate the effect of magnetic field on Cu-water nanofluid flow in non-parallel walls. Zeinali Heris et al. [40] studied nanofluid of CuO nanoparticles and distilled water has been prepared and its heat transfer characteristics have been studied through square cupric duct in laminar flow under uniform heat flux. Sahooli et al. [41] studied CuO/water nanofluid was synthesized by using polyvinylpyrolidone as the dispersant. Hooshyar et al. [42] studied rheological behaviour of ethylene glycol based nanofluids containing maghemtite nanoparticles.

To the best of authors' knowledge there is no any investigation to address the slip effects of stagnation viscous flow of nanofluids and heat transfer over a stretching sheet effect of magnetic field. The aim of the present study is to extend the work of Nadeem and Rizwan Ul Haq [35]. It is noticed that the boundary layer flow and heat transfer of a nanofluids with exhibits slip conditions and transformed the governing partial equations into ordinary differential equations solved numerically by using Keller-box method and compared the homotopy analysis method.

2.MATHEMATICAL FORMULATION

Consider two dimension problem with coordinate system in which the x-axis is taken as horizontally and y-axis is perpendicular to it. We consider steady incompressible flow of a nanofluid past a stretching surface coinciding with the plane $y = 0$ and flow is taken place at $y \geq 0$, where y is the coordinate measured normal to the stretching surface. Here we assume the sheet is stretched with the free stream velocity is in the form $u_e = bx$ and the velocity of the stretching sheet $u_{W=}ax$, where a, b are positive constants and x is coordinate measured along the stretching surface. A constant magnetic field of strength B_0 is applied parallel to the y-axis and the induced magnetic field assumed to be negligible. The temperature T and the nanoparticle volume fraction C take constant value T_f and C_W, respectively at the wall and the constant value, T_∞ and C_∞, respectively far away from the wall. Moreover slip conditions are taken into the account at wall and the effect of stagnation has been considered when fluid is passing through the stretching sheet. The basic continuity, momentum, energy and concentration equations for the present boundary layer flow are reduced to the following equations ([43]).

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \,, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e(x)\frac{du_e(x)}{dx} + \upsilon\frac{\partial^2 u}{\partial y^2} + \frac{\sigma {B_0}^2}{\rho_f}(u_e - u), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+$$

$$\tau\left\{D_B\frac{\partial \emptyset}{\partial y}\frac{\partial T}{\partial x}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\}, \qquad (3)$$

$$u\frac{\partial \emptyset}{\partial x}+v\frac{\partial \emptyset}{\partial y}=D_B\frac{\partial^2 \emptyset}{\partial y^2}+\left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2}. \qquad (4)$$

where u and v denote the respective velocities in the x- and y-directions respectively, where p is the pressure, ρ_f is the density of the base fluid, α is the thermal diffusivity, v is kinematic viscosity, D_B the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient $\tau=\frac{(\rho c)_p}{(\rho c)_f}$ is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid with ρ being the density, c is the volumetric volume expansion coefficient and ρ_p is the density of the particles. The corresponding boundary conditions are

$$u=u_w(x)+N_1\frac{\partial u}{\partial y},\ v=0,\ K_f\frac{\partial T}{\partial y}=h_f\left(T_f-T\right),\ C=C_w\ at\ y=0$$

$$u=u_e(x), v\to 0, T\to T_\infty, C\to C_\infty\ as\ y\to\infty \qquad (5)$$

Where, K_f is the thermal conductivity of the fluid, h_f is the convective heat transfer coefficient, T_f is the convective fluid temperature below the moving sheet. Using Roseland approximation of radiation for an optically thick layer

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \qquad (6)$$

Where $\sigma^*$is the Stefan-Boltzmann constant, k^* is the absorption coefficient. Assuming that the temperature difference within the flow is such that T^4 may be expanded in a Taylor series and expanding T^4 about T_∞, the free stream temperature and neglecting higher orders we get

$T^4\equiv 4T_\infty^3T-3T_\infty^4$.

Introducing the following similarity transformations

$$\psi=(av)^{1/2}xf(\eta),\qquad \eta=\left(\frac{a}{v}\right)^{1/2}y,$$

$$u=axf'(\eta),\qquad v=-\sqrt{av}\,f(\eta),$$

$$\theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty},\qquad \phi(\eta)=\frac{C-C_\infty}{C_W-C_\infty} \qquad (7)$$

Eq. (1) is satisfied identically and Eqs. (2-5) take the following forms

$$f'''-f'^2+ff''+A^2-M(f'-A)=0 \qquad (8)$$

$$\left(1+\frac{4}{3}R\right)\theta''+\Pr\left(f\theta'+Nb\phi'\theta'+Nt\theta'^2\right)=0, \qquad (9)$$

$$\phi''+Le\,Pr(f\phi')+\frac{Nt}{Nb}\theta''=0. \qquad (10)$$

Transformed boundary conditions are

$$f(0)=0, f'(0)=1+\lambda f''(0),\quad \theta'(0)=-\gamma[1-\theta(0)]\text{ and }\phi(0)=1$$

$$f'(\infty)=A, \theta(\infty)=0, \phi(\infty)=0. \qquad (11)$$

Where, the prime denotes differentiation with respect to η.

$$M=\frac{\sigma B_0^2 x}{\rho a}, Nb=\frac{\rho c_p\ D_B\,(\phi_W-\phi_\infty)}{\rho c_f V}, \Pr=\frac{V}{\alpha}, \text{Le}=\frac{V}{D_B}, Nt=\frac{\rho c_p\ D_T\,(T_f-T_\infty)}{\rho c_f\,V\,T_\infty}, R=\frac{4\sigma^* T_\infty^3}{KK^*},\ \lambda=N_1\sqrt{\frac{a}{v}},\qquad \gamma=\left({}^{h_f}\!\big/_{k_f}\right)\sqrt{\frac{v}{a}},$$

$$A=b/a. \qquad (12)$$

Here M, Nb, Nt, Pr, Le, R, λ and γ are denote the magnetic parameter, Brownian motion parameter, the Thermophoresis parameter, the Prandtl number, the Lewis number, the velocity slip parameter and the Biot number, respectively. Expressions for the Skin friction, Nusselt number Nu_x and the Sherwood number Sh_x are

$$C_{fx}=\frac{\mu_f}{\rho u_w^2}\left[\frac{\partial u}{\partial y}\right]_{y=0}, Nu_x=\frac{xq_w}{k\left(T_f-T_\infty\right)}, Sh_x=\frac{xq_m}{D_B\left(C_w-C_\infty\right)}, \qquad (13)$$

Where q_w and q_m are the heat flux and mass flux, respectively

$$q_w=-\left[\frac{\partial T}{\partial y}\right]_{y=0}, q_m=-D_B\left[\frac{\partial C}{\partial y}\right]_{y=0} \qquad (14)$$

Substituting Eq. (6) in Eqs. (12)-(13), we obtain,

$$Re_x^{1/2}C_{fx}=f''(0),\quad Re_x^{1/2}C_{fx}=-\theta'(0),\quad Re_x^{1/2}C_{fx}=-\phi'(0) \qquad (15)$$

Here $\mathrm{Re}_x=u_w x/\nu$ is the local Reynolds number.

3. METHOD OF SOLUTION

As (8)-(10) are nonlinear, it is impossible to get closed-form solutions. Consequently, the equations with the boundary conditions (11) are solved numerically by means of a finite-difference scheme known as the Keller-box method which has been found to be very suitable in dealing with nonlinear problems easily adaptable to solving equations of any order. The principal steps in the Keller box method is to get the numerical solutions are the following

i). Transform the given first-order equations of ODEs to a system;

ii). Write the reduced ODEs in finite differences;

iii). By using Newton's method, linearize the algebraic equations and write them in a vector form;

iv). Solve the linear system by the block tri-diagonal elimination technique.

4. SOLUTION PROCEDURE

4.1. Finite-Difference Method:

To solve these transformed differential Eqs. (8)-(10) subjected to the boundary conditions (11), Eqs. (8)-(10) are first converted into a system of seven first-order equations, and the difference equations are then expressed using central differences. For this purpose, we introduce new dependent variables $p(\eta)$, $q(\eta)$, $\theta(\eta)$, $t(\eta)$, $\phi(\eta)$ and $v(\eta)$

$$f' = p, p' = q, \theta' = t, \phi' = v \qquad \text{(a)}$$

so that Eqs. (8)-(10) can be written as

$$q' + fq - \frac{2n}{n+1}p^2 - Mp = 0 \qquad \text{(b)}$$

$$\frac{1}{Pr}\left(1 + \frac{4}{3}R\right)t' + ft + Nb(tv) + Nt(t^2) + Ec(q^2) = 0 \qquad \text{(c)}$$

$$v' + Le\,(fv) + \frac{Nt}{Nb}t' - \gamma\phi = 0 \qquad \text{(d)}$$

Terms of the new dependent variables, the boundary conditions (11) are given by

$$p(0) = 1,\ f(0) = 0,\ \theta(0) = 1,\ \phi(0) = 1,$$
$$p(\eta) \to 0,\quad \theta(\eta) \to 0,\quad \phi(\eta) \to 0 \text{ as } \eta \to \infty \qquad \text{(e)}$$

We now consider the segment η_{j-1},η_j with $\eta_{j-1/2}$ as the midpoint, which is defined as below:

$$\eta_0 = 0,\quad \eta_j = \eta_{j-1} + h_j,\ \eta_J = \eta_\infty$$

Where h_j is the $\Delta\eta$-spacing and=1, 2... J is a sequence number that indicates the coordinate location.

4.2. Newton's Method:

To, linearize the nonlinear system, we use Newton's method, by introducing the following expression:

$$f_j^{(k+1)} = f_j^{(k)} + \delta f_j^{(k)},\ p_j^{(k+1)} = p_j^{(k)} + \delta p_j^{(k)},\quad q_j^{(k+1)} = q_j^{(k)} + \delta q_j^{(k)},$$
$$\theta_j^{(k+1)} = \theta_j^{(k)} + \delta\theta_j^{(k)},\ t_j^{(k+1)} = t_j^{(k)} + \delta t_j^{(k)},\quad \phi_j^{(k+1)} = \phi_j^{(k)} + \delta\phi_j^{(k)},$$
$$v_j^{(k+1)} = v_j^{(k)} + \delta v_j^{(k)} \qquad \text{(f)}$$

Where k=0, 1, 2, ... We then insert the left-hand side expressions in place of $\delta f^{(k)}$, $\delta p^{(k)}$,$\delta q^{(k)}$, $\delta\theta^{k}$, ,$\delta t^{(k)}$,$\delta\phi^{(k)}$ and $\delta v^{(k)}$.

4.3. Block-elimination Method:

The linearized difference equations can be solved by the block-elimination method as outlined by Cebeci and Bradshaw [44], since the system has block-tridiagonal structure. Commonly, the block-tridiagonal structure consists of variables or constants, but here an interesting feature can be observed that it consists of block matrices. In a matrix-vector form, can be written as

$$A\delta = r \qquad \text{(g)}$$

Where

$$A = \begin{bmatrix} [A_1] & [C_1] & & & & \\ [B_2] & [A_2] & [C_2] & & & \\ & & \ddots & & & \\ & & \ddots & & & \\ & & \ddots & & & \\ & & & [B_{J-1}] & [A_{J-1}] & [C_{J-1}] \\ & & & & [B_J] & [A_J] \end{bmatrix}$$

$$\delta = \begin{bmatrix} [\delta_1] \\ [\delta_2] \\ \vdots \\ [\delta_{J-1}] \\ [\delta_J] \end{bmatrix} \text{ and } r = \begin{bmatrix} [r_1] \\ [r_2] \\ \vdots \\ [r_{J-1}] \\ [r_J] \end{bmatrix}$$

This block tridiagonal matrix can be solved by LU method.

5. RESULTS AND DISCUSSIONS:

The model for nanofluids passing a slip boundary convective stretching sheet nano energy conversion thermal system has been presented in this study. Present nano energy conversion effects of dimensionless parameters including, magnetic parameter (M), Prandtl number (Pr), Brownian diffusion coefficient (Nb), thermophoresis diffusion coefficient (Nt), stagnation parameter (A), slip parameter (λ), Biot number (γ) are the main interests of the study. A similarity transformation has been used to convert the non-linear, coupled partial differential equations to a set of non-linear, coupled ordinary differential equations. Keller Box method has been used to obtain solutions of those equations. A comparison of the Homotopy Analysis method and Numerical approximate solutions of the Nusselt number and Sherwood number for varying fixed parameters are shown in graphs and tables. Numerical solutions of the skin friction, heat transfer and mass transfer are presented for varying stagnation and velocity slip parameter values at different iterations. The effect of the stagnation parameter is seen to enhance skin friction, the heat transfer rate and mass transfer rate.

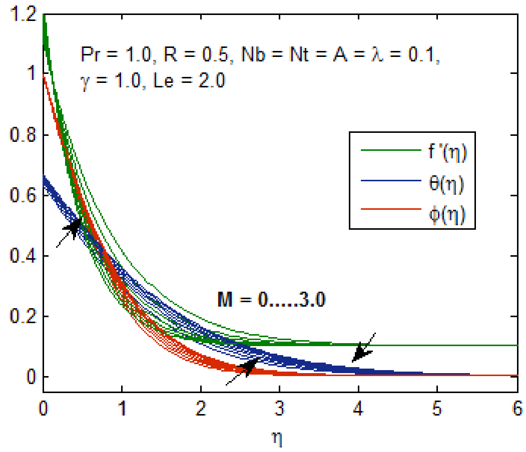

Figure 1. *Effect of M on velocity, temperature and concentration profiles.*

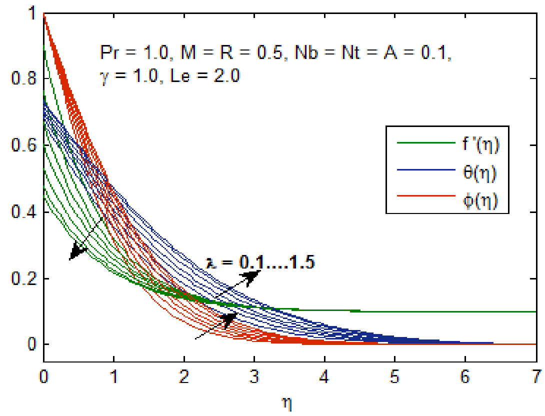

Figure 2. *Effect of λ on velocity, temperature and concentration profiles.*

Similarly the velocity slip parameter reduces the skin friction, heat transfer rate and mass transfer rate. Fig. 1 shows the effect of magnetic parameter on the velocity, temperature and concentration profiles. It is observed that the dimensionless velocity decreases with increasing values of the magnetic parameter M. It is noticed that the temperature profile increases with increasing values of the magnetic parameter M. This is due to the increase in thermal conductivity with an increase in the volume fraction of nanoparticles. It is also noticed from Fig. 1, that the wall temperature decreases with an increase in the magnetic field. As the magnetic parameter M increases, the nanoparticle concentration also increases. It is due to the fact that transverse magnetic field creates a drag (Lorentz force) which resists the flow and decreases the wall temperature.

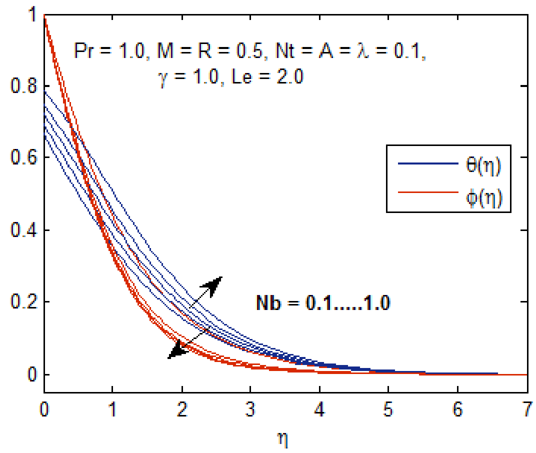

Figure 3. *Effect of Nb on temperature and concentration profiles.*

Fig. 2 illustrates the influence of slip parameter in stretching sheet on the velocity, temperature and concentration. It depicts that increasing the values of (λ) decreases the velocity profile f(η) for stretching sheet. When we increase slip parameter (λ) (see Fig. 2), there will be more resistance between fluid and sheet that causes increase in temperature profile and increase in thermal boundary layer thickness. In the same way, increase the values of (λ) increases the velocity profile for stretching sheet. However, the velocity boundary layer thickness shows a decreasing function of slip parameter while the thermal and concentration boundary layer thickness shows an increasing function of slip parameter. The effects of Brownian motion parameter (Nb) and thermophoresis parameter (Nt) on the temperature and concentration profiles for selected values of parameters are shown in Figs. 3 and 4, respectively. From Fig. 3 we see the Brownian motion effect Nb on the temperature distribution and concentration of nanoparticle, respectively. It is noticed that the temperature distribution and thermal boundary layer thickness enhance with larger values of Nb. Physical reason is that the kinetic energy of the nanoparticle increases due to the strength of this chaotic motion and as a result the fluid's temperature increases, whereas the opposite trend is seen on the concentration nanoparticle as depicted in same figure. It can be seen that the concentration nanoparticle decreases due to the increasing values of Nb.

It can be concluded that the Brownian motion parameter makes the fluid warm within the boundary and at that time aggravates deposition particles away from the regime of fluid to the surface that causes a decrease in the concentration profile as well as the thickness of boundary layer.

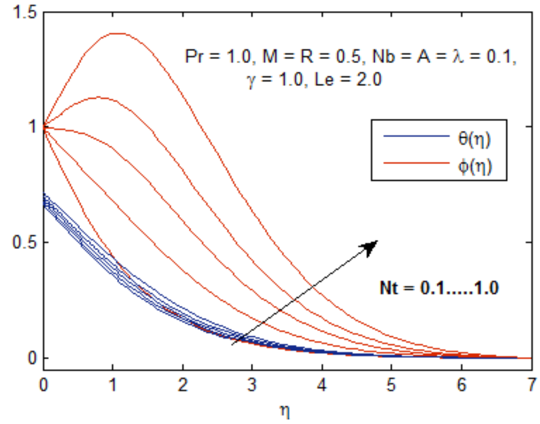

Figure 4. *Effect of Nt on temperature and concentration profiles.*

The larger values of Brownian motion imply the strong behavior for the smaller particle, whereas for stronger particle the smaller values of Nb are applied.

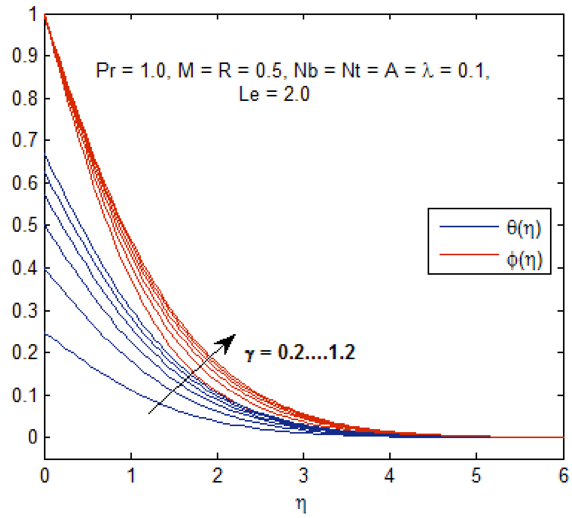

Figure 5. *Effect of* (γ) *on temperature and concentration profiles.*

As the parameter Nt increase, the temperature at a point increases. As a consequence, the thickness of the thermal boundary layer increases with the increase of Nt. An increase in Nt results in an increase of the temperature difference between the sheet and the ambient fluid and consequently the thermal boundary layer thickness increases. In the same way as increasing the thermophoresis parameter, concentration also increased (see fig. 4). Fig. 5 illustrates the influence of Biot number in stretching sheet on temperature and concentration profiles. It is noticed that an increase in (γ) leads to an increase in the temperature and also the concentration profile increases with the increasing value of Biot number. The effect of Lewis number (Le) on the dimensionless rescaled nanoparticle volume fraction is shown in Fig. 6. It is observed that the concentration boundary layer thickness decreases with increasing Le. This is due to the decrease in mass diffusivity or the Brownian motion of the nanoparticle. Fig. 7 shows the effect of stagnation parameter (A) on velocity profile. It is observed Stagnation parameter for the stretching case forces the fluid to move speedily so when we increase stagnation parameter A velocity field increases rapidly and the boundary layer thickness also increases. Fig. 8 shows the nature of heat transfer coefficient against magnetic parameter M for three values of velocity slip parameter. It is observed that rate of heat transfer decreases with magnetic and velocity slip parameters. Fig. 9 displays the nature of heat transfer coefficient against Thermophoresis parameter Nt for three values of Brownian motion Nb parameter.when the value of Brownian motion parameter Nb is increased from 0.1 to 0.5.The reduced nusselt number effectively decline due to thermophoresis parameter Nt.This reduction occurs because of the nanoparticles movement from stretching wall to the quiescent fluid.

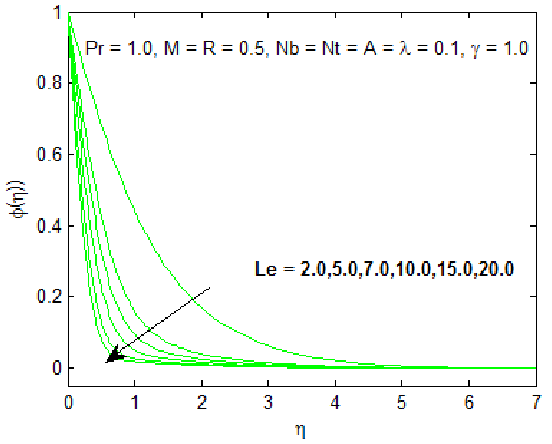

Figure 6. *Effect of Lewis number (Le) on concentration profiles.*

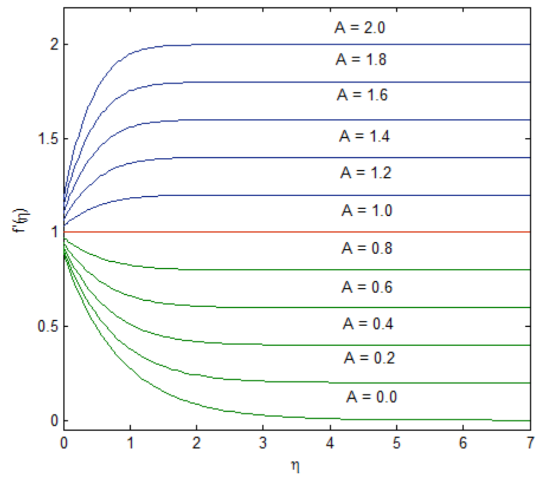

Figure 7. *Effect of velocity ratio parameter (A) on velocity profile.*

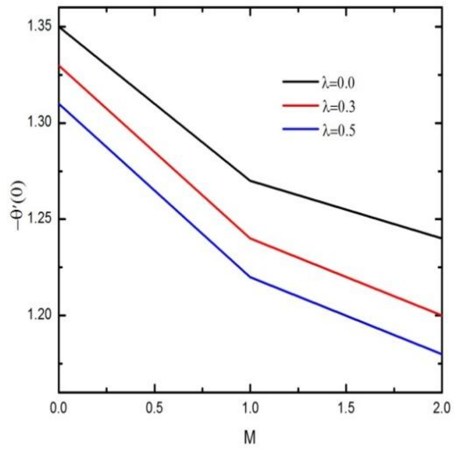

Figure 8. *Variation of heat transfer rate with magnetic parameter (M) and velocity slip parameter (λ).*

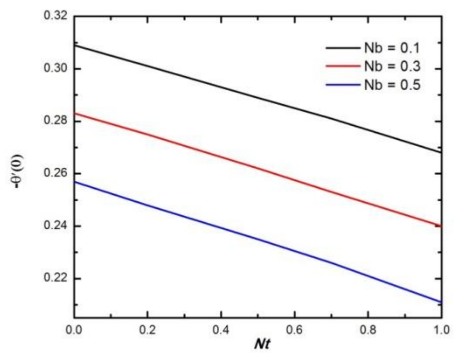

Figure 9. *Variation of heat transfer rate with Thermophoresis (Nt) and Brownian motion(Nb) parameters.*

Table-1. *Comparison table for heat and mass transfer rates for Pr, Nb, Nt, Le parameters.*

Pr	Nb	Nt	Le	$-\theta'(0)$		$-\phi'(0)$	
				Nadeem and Rizwan Ul Haq [35]	Present work	Nadeem and Rizwan Ul Haq [35]	Present work
1.0	*0.1*	*0.1*	*2.0*	*0.2877*	*0.2789*	*0.7487*	*0.7467*
1.5				*0.2963*	*0.2960*	*0.7477*	*0.7465*
2.0				*0.3048*	*0.3013*	*0.7477*	*0.7443*
2.5				*0.3132*	*0.3031*	*0.7457*	*0.7454*
	0.2			*0.3061*	*0.3050*	*0.8106*	*0.8088*
	0.3			*0.2989*	*0.2975*	*0.8323*	*0.8298*
	0.4			*0.2916*	*0.2910*	*0.8431*	*0.8410*
		0.3		*0.2891*	*0.2870*	*0.8155*	*0.8055*
		0.4		*0.2866*	*0.2834*	*0.7886*	*0.7797*
		0.5		*0.2840*	*0.2798*	*0.7625*	*0.7610*
			3.0	*0.2824*	*0.2812*	*0.9105*	*0.9100*
			4.0	*0.2809*	*0.2804*	*1.0655*	*1.0588*
			5.0	*0.2793*	*0.2765*	*1.2274*	*1.2256*

Table 2. *Calculation of skin friction coefficient, Nusselt and Sherwood number for various values of A and λ when $M = 1.0,\ Nb = Nt = 0.1, \gamma = 0.6, R = 0.5, Pr = 1.0, Le = 2.0$*

A	λ	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.0	*0.1*	*1.20564*	*0.25039*	*0.60921*
0.2		*1.03434*	*0.27785*	*0.71750*
0.5		*0.70546*	*0.30300*	*0.83798*
2.0		*1.86403*	*0.36610*	*1.27350*
3.0		*4.18668*	*0.38930*	*1.50593*
0.1	*0.1*	*1.12526*	*0.26603*	*0.66841*
	0.2	*0.98386*	*0.25892*	*0.63347*
	0.3	*0.87618*	*0.25295*	*0.60535*
	0.4	*0.79104*	*0.24785*	*0.58208*
	0.5	*0.72180*	*0.24341*	*0.56242*

6. CONCLUSIONS

The problem of MHD boundary layer flow of nanofluids and heat transfer over a stretching sheet with stagnation and slip convective boundary conditions at the wall has been investigated numerically. A similarity solution is presented which effects, magnetic, Brownian motion and thermophoresis parameters, chemical reaction and radiation parameter. The governing equations associated to the boundary conditions were transformed to a two point non-linear ODEs with the help of similarity transformation equations. The solutions of the problem were numerically obtained using the Keller box method. The following conclusions are deduced:

- The velocity profile increases, whereas dimensionless temperature and concentration are increases with magnetic parameter.
- The dimensionless temperature and concentration are increases with Thermophoresis parameter.
- The dimensionless temperature profile increases and concentration are decreases with Brownian motion parameter.
- The temperature increases with viscous dissipation parameter and the concentration decreases with Lewis number.
- The heat transfer rate decreases with magnetic parameter and velocity slip parameter.
- The heat transfer rate decreases with Thermophoresis and Brownian motion parameters.

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