Magnetic Properties in a Spin-1 Random Transverse Ising Model on Square Lattice

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(Received: 10 May 2011 and Accepted: 25 May 2011)

Abstract:
In this paper we investigate the effect of a random transverse field, distributed according to a trimodal distribution, on the phase diagram and magnetic properties of a two-dimensional lattice (square with z=4), ferromagnetic Ising system consisting of magnetic atoms with spin-1. This study is done using the effective-field theory (EFT) with correlations method. The equations are derived using a probability distribution method based on the use of exact Van der Waerden identities. We present our numerical results, such as the phase diagrams, the thermal variations of the transverse magnetization, the internal energy, the magnetic specific heat as a function of different values of p, the concentration of the random transverse field. As a result, the critical values of transverse field , temperature and concentration were obtained for the square lattice.
Keywords: *Transverse Ising model, Effective-field theory, Phase transition, Surface magnetic properties*

1. INTRODUCTION

Recently, a problem of growing interest is associated with the random transverse field Ising model (RTFIM). One of the interesting phenomena in the RTFIM is the occurrence of a tricritical behavior. The RTFIM has examined by the use of various techniques, such as mean field theory, Monte-Carlo simulations, renormalization group calculations, Bethe-Pierls approximation and effective-field theories (EFT) [1-6]. Lately, some interest has been directed to the understanding of more complicated systems in the presence of random fields, i.e. the transverse Ising model, the amorphous Ising ferromagnet, the Blume-Capel model and spin-S Ising model.

Section 2, described the EFT with correlations and the formulation of EFT is applied to a square lattice (z=4) consisting of spin-1 atoms. In section 3, we present our numerical results and discussions, such as the phase diagrams, the thermal variations of the transverse magnetization, the internal energy, the magnetic specific heat as a function of different values of p, the concentration of the random transverse field.

2.THEORY AND FORMULATION

The surface random transverse field Ising model is described by the following Hamiltonian:

$$H = -J_S \sum_{\langle ij \rangle} S_i^z S_j^z - \sum_i (\Omega_S)_i S_i^x \tag{1}$$

where S_i^z, S_i^x are components of a spin-1 operator at site i, J_S is the exchange interaction at the surface. The first and second summations are over nearest-neighbor sites and single sites located on the free space, respectively. The transverse field

$(\Omega_S)_i$ is assumed to have the trimodal probability distribution:

$$P((\Omega_S)_i) = p\delta((\Omega_S)_i) + \frac{(1-p)}{2}\left[\delta((\Omega_S)_i - \Omega_S) + \delta((\Omega_S)_i + \Omega_S)\right] \quad (2)$$

where p denoted the fraction of spins not exposed to the transverse field Ω_S. Because of randomness of transverse fields, we should perform the random average of $(\Omega_S)_i$ with using the probability distribution function $P((\Omega_S)_i)$. We can define $\mu^z = \langle\langle S_i^z \rangle\rangle_r$ and $\mu^x = \langle\langle S_i^x \rangle\rangle_r$ and $\bar{\eta}^2 = \langle\langle (S_i^z)^2 \rangle\rangle_r$, where $\langle ... \rangle_r$ denotes the random average and $\langle ... \rangle$ indicates the canonical thermal average. Thus, doing the random average, we can obtain:

$$\mu^z = \left[\cosh(\bar{\eta} J_S \nabla) + \frac{\mu^z}{\bar{\eta}} \sinh(\bar{\eta} J_S \nabla)\right]^z \bar{F}_S(x)\Big|_{x=0}, \quad (3)$$

$$\mu^x = \left[\cosh(\bar{\eta} J_S \nabla) + \frac{\mu^z}{\bar{\eta}} \sinh(\bar{\eta} J_S \nabla)\right]^z \bar{G}_S(x)\Big|_{x=0}, \quad (4)$$

$$\bar{\eta}^2 = \left[\cosh(\bar{\eta} J_S \nabla) + \frac{\mu^z}{\bar{\eta}} \sinh(\bar{\eta} J_S \nabla)\right]^z \bar{H}_S(x)\Big|_{x=0}, \quad (5)$$

$$\bar{F}_S(x) = \int P((\Omega_S)_i) F_S(x) d((\Omega_S)_i)$$

$$\bar{G}_S(x) = \int P((\Omega_S)_i) G_S(x) d((\Omega_S)_i)$$

$$\bar{H}_S(x) = \int P((\Omega_S)_i) H_S(x) d((\Omega_S)_i) \quad (6)$$

and for spin-1, we have:

$$F_S(x) = \frac{x}{\sqrt{B^2 + x^2}} \frac{2\sinh(\beta\sqrt{B^2 + x^2})}{1 + 2\cosh(\beta\sqrt{B^2 + x^2})}$$

$$G_S(x) = \frac{B}{\sqrt{B^2 + x^2}} \frac{2\sinh(\beta\sqrt{B^2 + x^2})}{1 + 2\cosh(\beta\sqrt{B^2 + x^2})}$$

$$H_S(x) = \frac{B^2 + (x^2 + y^2)\cosh(\beta y)}{y^2\left[1 + 2\cosh(\beta y)\right]} \quad (7)$$

where $y = \sqrt{B^2 + x^2}$

The random averaged internal energy $\bar{U}$ is given by:

$$\frac{\bar{U}}{N} = -\langle\langle A_i S_i^z \rangle\rangle_r - \langle (\Omega_S)_i \langle S_i^z \rangle\rangle_r \quad (8)$$

where N is the number of magnetic atoms. It is clear that for the evaluation of $\bar{U}$, we must know μ^z, μ^x and $\bar{\eta}$. Then these quantities can be easily obtained by solving (3)-(5) numerically. The averaged magnetic specific heat can be written as:

$$\bar{C} = \frac{\partial \bar{U}}{\partial T}, \quad (9)$$

3. RESULTS AND CONCLUSION

From the Figure 1, we can see that the effect of a random transverse Ising model. Namely, the critical temperature gradually decreases from its Ising value at $\Omega = 0$ and rapidly vanishes when the transverse field approaches some critical Ω_c depending on the value of *p*. When the transverse field is bi-modally distributed (*p=0*), the critical temperature gradually decrease from its value $T_c(\Omega = 0)$ ($T_c / J_S = 2.011776$), to vanish at some critical value Ω_c ($\Omega_c / J_S = 3.01985$).

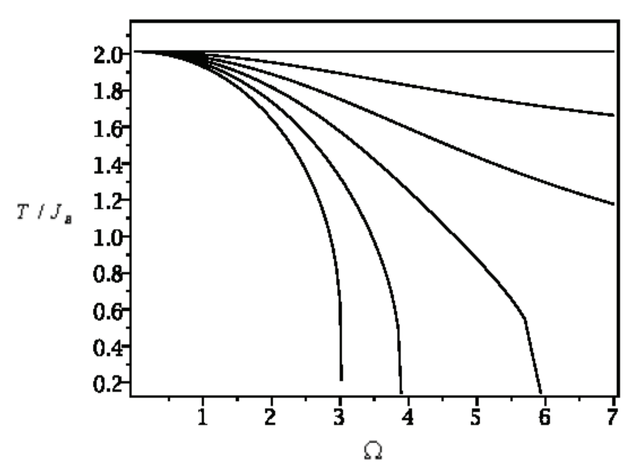

Figure 1: *The phase diagram (T / J_S - Ω) for various p(from left to right, p=0,0.2,0.4,0.6,0.8,1) in the square lattice*

In Figures 2 and 3, the temperature dependences of the transverse magnetizations as well as the

From the limit behaviors obtained for $p \cong 0$ and $p \cong 1$, one can reasonably expect that there appears a critical value p^* of the concentration of zero transverse field sites (p^*=0.657296) showing two different behaviors of the system: for $0 \le p \le p^*$, the cluster of zero transverse field sites is small and hence the order, at T=0, is destroyed beyond a finite critical value Ω_C and for $p^* \le p \le 1$, such a cluster is sufficiently large to keep order in the system at very low temperature, even in the limit of infinitely large values of the transverse field.

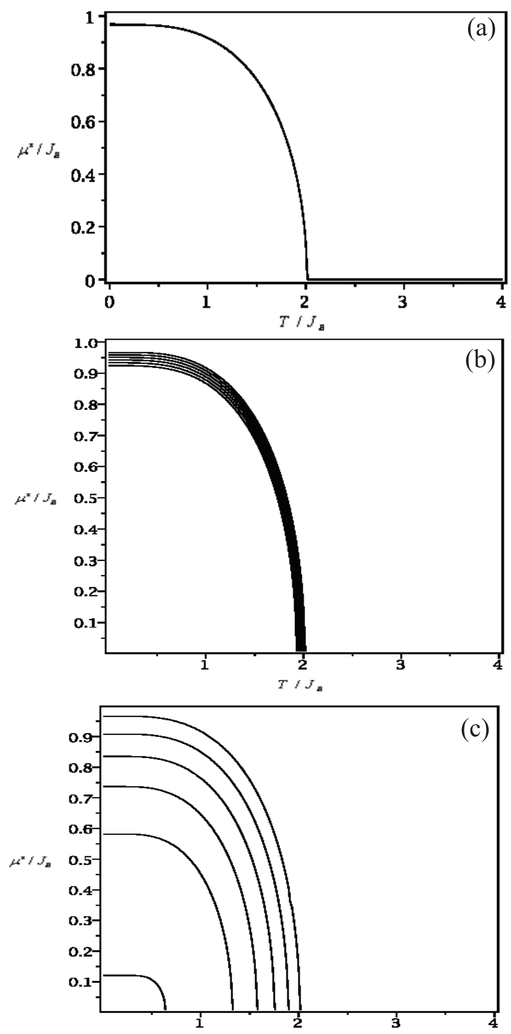

Figure 2: *μ^z / J_S -T/J_S for various p (from left to right p=0,0.2, 0.4, 0.6,0.8,1)and (a): Ω =0.1,(b): Ω =1 and (C): Ω =3.*

parameter $q = \overline{\eta}^2$ are plotted for the square lattice, when the transverse field is fixed ((a): Ω=0.1, (b): Ω=1, (c): Ω=3).

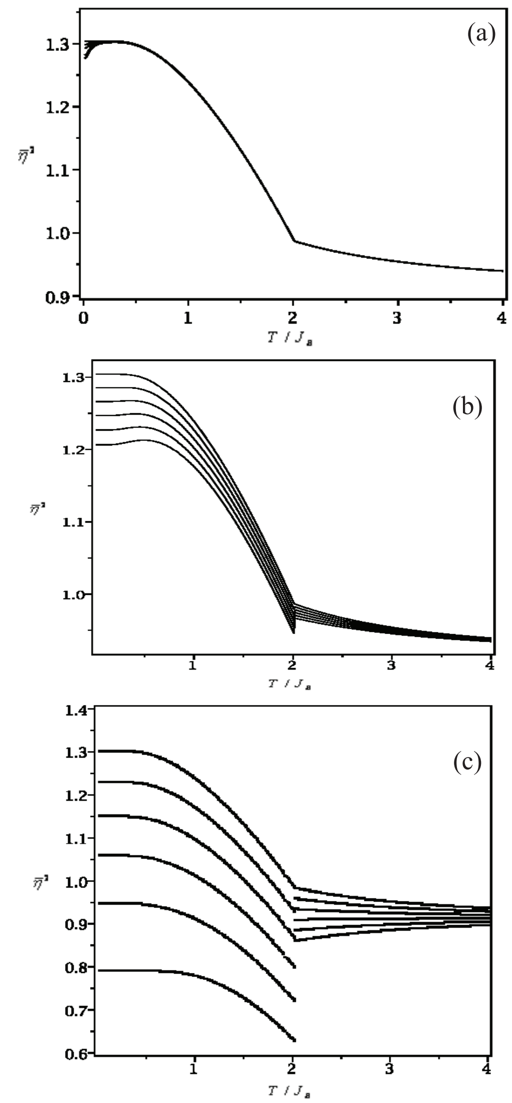

Figure 3: *$\overline{\eta}^2$ - T/J_S for various p (from down to up p=0, 0.2,0.4, 0.6,0.8,1) and (a): Ω =0.1,(b): Ω =1 and (C): Ω =3.*

Finally, in Figures. 4, 5, the temperature dependences of the internal energy ($\frac{-\overline{U}}{NJ_S}$) and the magnetic specific heat ($\frac{\overline{C}}{k_B N}$) for various p in the same system are plotted. We can see that, if the transverse field increases, then the absolute value of internal energy in the systems increases. On the other hand, the magnetic specific heat is gradually depressed by increasing the transverse field strength Ω. It can also be seen that the jump at the critical temperature gradually disappears with the increasing value of Ω.

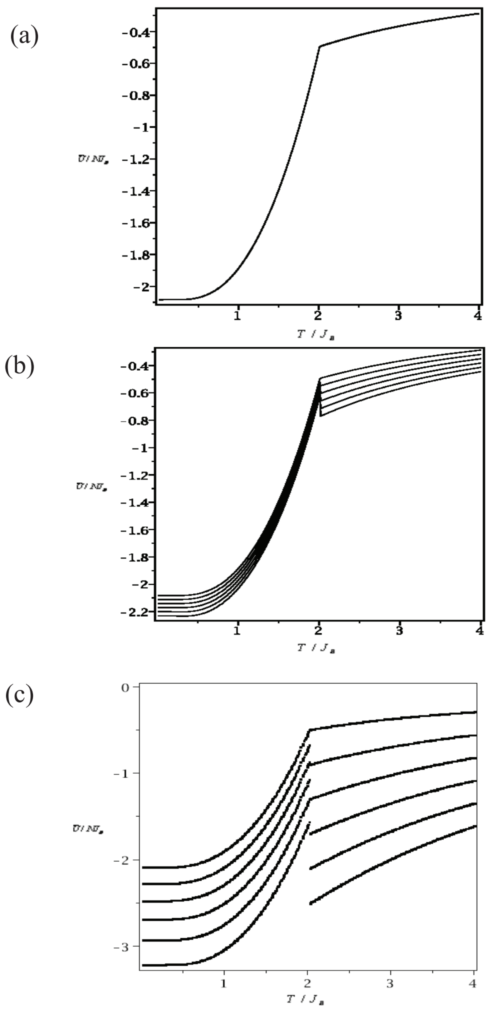

Figure 4: *($\bar{U} / NJ_S$)-T/J_S for various p (from down to up p=0,0.2,0.4,0.6,0.8,1) and (a):* Ω *=0.1, (b):* Ω *=1 and (C):* Ω *=3.*

Moreover, we can see that, by increasing the the transverse field strength Ω, the transverse magnetization μ^z, the parameter $q = \bar{\eta}^2$, the internal energy $\bar{U}$ and the magnetic specific heat $\bar{C}$ are gradually separated for various *p*, and critical temperatures T_C / J_S are rapidly decreased for smaller *p*.

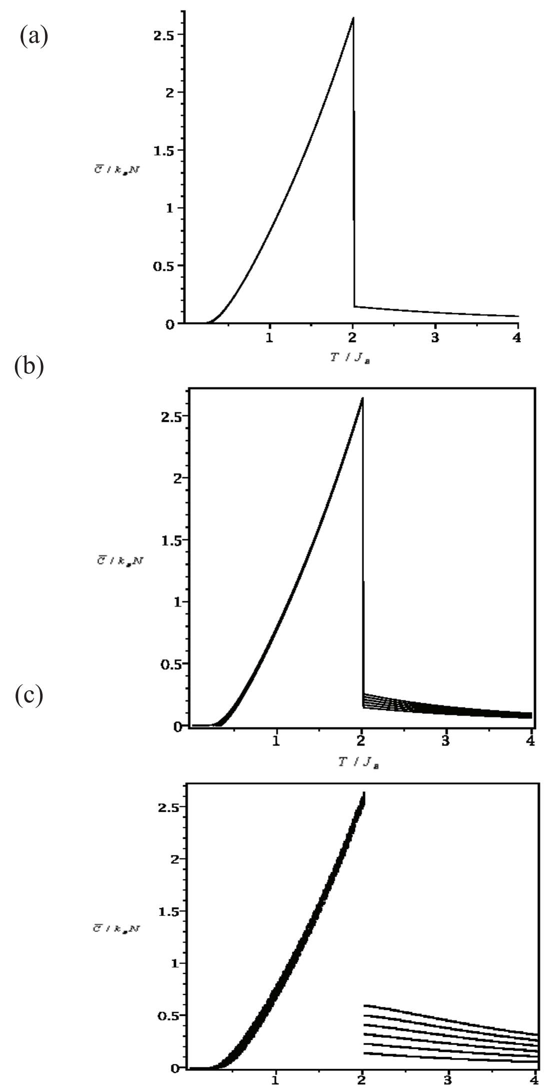

Figure 5: *($\bar{C} / k_B N$)-T/J_S for various p (from down to up p=0 ,0.2,0.4,0.6,0.8,1) and (a):* Ω *=0.1, (b):* Ω *=1 and (C):* Ω *=3.*

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