Int. J. Nanosci. Nanotechnol., Vol. 14, No. 3, Sept. 2018, pp. 177-185

Sound Wave Propagation in Viscous Liquid-Filled Non-Rigid Carbon Nanotube with Finite Length

Sedigheh Basirjafari

Department of Acoustics and Audio Engineering, IRIB University, P.O. Box 1986916511, Tehran, I.R. Iran.

(*) Corresponding author: basirjafari@iribu.ac.ir
(Received: 23 August 2016 and Accepted: 09 January 2018)

Abstract

In this paper, numerical results obtained and explained from an exact formula in relation to sound pressure load due to the presence of liquid inside the finite-length non-rigid carbon nanotubes (CNTs), which is coupled with the dynamic equations of motion for the CNT. To demonstrate the accuracy of this work, the obtained formula has been compared to what has been used by other researchers. For this purpose, the solution of the modified complex Helmholtz equation was derived by considering the non-rigidity of the CNT and the wave reflections at the open ends of the CNT for three different liquids with or without considering the relaxation time. The results showed that neglecting the non-rigidity of CNT would cause a decrease on the pressure fluctuations and the pressure associated with the viscosity force of the liquid in the liquid-filled CNT, at both axi-symmetric, and asymmetric cases. Also, it is showed that the viscous liquid in a CNT is a dispersive medium for sound wave propagation and ignoring the energy loss in the liquid in the high frequency analysis and ignoring the non-rigidity of the CNT would cause errors in the prediction of the sound pressure load exerted on the finite-length liquid-filled CNT.

Keywords: *Finite-length liquid-filled CNT, Viscosity force, Modified complex Helmholtz equation, Non-rigidity, Sound pressure load.*

1. INRODUCTION

The acoustical transition behavior of CNTs could be excessively sensitive to even very small deformations of their geometry [1-3]. Due to this property, the study on vibrational characteristics of wave propagation in CNTs under real conditions without facile assumptions is very important for designing CNT-based nanocomposites [4-11] and nano-devices. Owing to nanometer dimensions of CNTs, it is difficult to set up controlled experiments to measure the properties of an individual CNT. The nonlocal-elastic beam [12-21], plate [22, 23] and shell [24-28] models could predict the vibrational behavior of the CNTs much better than the traditional elastic shell [29-36] models.

In previous studies, in order to obtain an analytical solution, the cylindrical shell has been assumed as an infinite-length shell which would cause an error in the vibrational analysis and wave propagation when it is finite in length, particularly as the length of the shell is equivalent to its radius [37]. To the author's knowledge, there have been no studies on sound wave propagation in finite-length CNTs where reflective waves from two open ends of the CNTs are being taken into account [29, 30, 38] except that of the author's own work [28]. Another point that has not been formally researched on is the non-rigidity of the CNT [25, 26, 29-32, 38-41]. In this paper, the non-rigidity effect of the carbon nanotube (CNT) on sound pressure is studied through considering finite acoustical impedance on the wall of the CNTs. Due to the fact that the vibrational frequencies of the CNT are approximately terahertz (THz) owed to its extremely small dimensions, the relaxation time effect must be considered through the dissipative wave equation due to the high frequency vibrational analysis needed for the liquid-filled CNT. Although, acoustic wave propagation in elastic tubes containing viscous fluid is not a new issue [42, 43], there have been only a few investigations on the wave propagation in viscous-fluid-conveying CNTs [38]. It is important to note that due to the properties of CNTs research on the existence of fluid within CNTs is essential especially in acoustic wave propagation applications.

In this paper, numerical results obtained and explained from an exact formula obtained from the author's related work as in [28] for sound pressure load through the existence of liquid inside the finite-length non-rigid carbon nanotubes (CNTs), which is coupled with the dynamic equations of motion for the CNT. The liquid relaxation time effect and the non-rigidity effect of the CNT are investigated in detail, for the liquid load acting inside the CNT. To study the sound wave propagation in CNTs under real conditions, the simplifying assumptions are avoided, as much as possible.

2. MODELLING THE FINITE-LENGTH LIQUID-FILLED NON-RIGID CNT

Liquid load has an important role in vibrational analysis of the liquid-filled CNT. To determine *f*, the sound pressure acted on the liquid-filled CNT due to the sound wave propagation must be calculated. In this study, numerical results obtained and explained from an exact formula obtained in [28] for sound pressure load due to the existence of liquid inside the finite-length non-rigid carbon nanotubes (CNTs) coupled with the dynamic equations of motion for the CNT. For this purpose, the non-rigid CNT is modeled as a thin cylindrical shell with radius R and thickness h in cylindrical coordinates (r,θ,x), where x is the coordinate in the axial direction, r is the radial direction, and θ is the circumferential direction (see Figure 1).

In [28], the author's obtained the "modified complex Helmholtz equation" ($\nabla^2 p+\bar{k}^2 p=0$) that describes the sound pressure in a viscous liquid (real media), in which ∇^2 is the Laplacian operator, p is the pressure of sound and $\bar{k}$ is a complex wave number that is given as

$$\bar{k}=\frac{\omega}{C_{\mathrm{f}}}\frac{1}{\sqrt{1-\mathrm{i}\omega\tau}} \tag{1}$$

in which $\tau=\left(4\eta/3+\eta_{\mathrm{B}}\right)\big/\rho_{\mathrm{f}}C_{\mathrm{f}}^2$ is called "relaxation time" [44] in which, η is the shear viscosity coefficient, η_{B} is the bulk viscosity coefficient, ρ_{f} is the equilibrium density of the media and C_{f} is the speed of sound in the media. Henceforth, in real media, sound wave propagation causes energy dissipation.

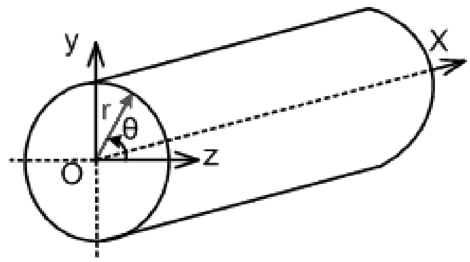

Figure 1. *Cylindrical coordinates.*

To determine the exact liquid load term due to the sound wave propagation inside the finite-length viscous-liquid-filled CNT, the solution of the modified complex Helmholtz equation is derived by considering the non-rigidity of the CNT and the wave reflections at the open ends of the CNT.

The complex wave number (Eq. (1)) can be rewritten as

$$\bar{k}=K+\mathrm{i}\zeta \tag{2}$$

where ζ is decay coefficient that describes energy dissipation.

Squaring of Eqs. (1) and (2) gives, respectively:

$$\bar{k}^2=\left(\frac{\omega}{C_{\mathrm{f}}}\right)^2\frac{1}{1-\mathrm{i}\omega\tau}$$

$$=\left(\frac{\omega}{C_{\mathrm{f}}}\right)^2\frac{1+\mathrm{i}\omega\tau}{1+(\omega\tau)^2} \tag{3}$$

and

$$\bar{k}^2 = K^2 - \zeta^2 + \mathrm{i}\,2K\zeta$$

or

$$\bar{k}^2 = A + \mathrm{i}\,B \tag{4}$$

in which

$$A \equiv K^2 - \zeta^2, B \equiv 2K\zeta \tag{5}$$

and complex conjugate of Eq. (4) gives

$$(\bar{k}^2)^* = A - \mathrm{i}\,B \tag{6}$$

A and B can be calculated from Eqs. (4) and (6) in terms of $\bar{k}^2$ and $(\bar{k}^2)^*$ as follows:

$$A = \frac{1}{2}\left[\bar{k}^2 + (\bar{k}^2)^*\right], \quad B = \frac{1}{2\mathrm{i}}\left[\bar{k}^2 - (\bar{k}^2)^*\right] \tag{7}$$

By substituting Eq. (3) and its complex conjugate into Eq. (6), Eq. (7) gives

$$A = \left(\frac{\omega}{C_\mathrm{f}}\right)^2 \frac{1}{1+(\omega\tau)^2}, \quad B = \left(\frac{\omega}{C_\mathrm{f}}\right)^2 \frac{\omega\tau}{1+(\omega\tau)^2} \tag{8}$$

By considering Eqs. (5) and (8), we have

$$K^2 - \zeta^2 = \left(\frac{\omega}{C_\mathrm{f}}\right)^2 \frac{1}{1+(\omega\tau)^2}, \quad 2K\zeta = \left(\frac{\omega}{C_\mathrm{f}}\right)^2 \frac{\omega\tau}{1+(\omega\tau)^2} \tag{9}$$

Therefore, K and ζ can be calculated from Eq. (9) as

$$K = \frac{\omega}{C_\mathrm{f}}\left\{\frac{\sqrt{1+(\omega\tau)^2}+1}{2\left[1+(\omega\tau)^2\right]}\right\}^{1/2}, \quad \zeta = \frac{\omega}{C_\mathrm{f}}\left\{\frac{\sqrt{1+(\omega\tau)^2}-1}{2\left[1+(\omega\tau)^2\right]}\right\}^{1/2}. \tag{10}$$

Since the amplitude of sound pressure wave in a real media decays as $\exp(-\zeta x)$, ζ is the absorption or decay coefficient. The real part of the complex wave number takes part in the wave phase velocity. The imaginary part of the complex wave number is equivalent to the sound wave absorption in the fluid. If $\omega\tau = 0$, Eq. (10) allows K to be equal to $k = \omega/C_\mathrm{f}$ where k is the free field wave number and ω is the angular frequency.

Now, one may compare the Helmholtz equation with the modified complex Helmholtz equation. As mentioned before, the vibrational frequencies of the CNT are in the range of THz because of its extremely small dimensions. In addition, the typical values of τ are about 10^{-12}s for all liquids except the highly viscous ones, like glycerin [44]. Thus, the assumption that $\omega\tau << 1$ fails for the frequency range of THz, and would cause an error in the sound wave propagation analysis of the liquid-filled CNT.

Figure 2 shows the non-dimensional wave number versus non-dimensional frequency of the modified Helmholtz equation. The real part of the non-dimensional wave number decreases by increasing $\omega\tau$. The imaginary part of the non-dimensional wave number initially increases by increasing $\omega\tau$ and then decreases by increasing $\omega\tau$.

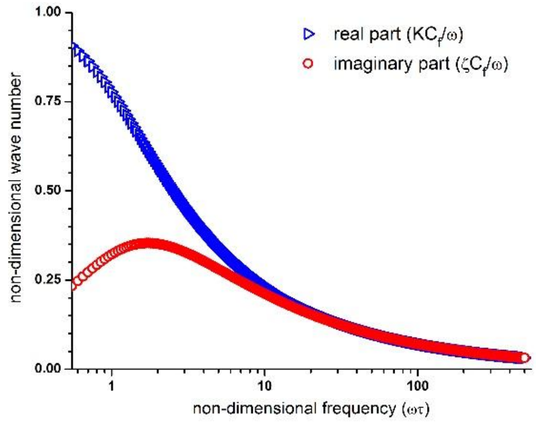

Figure 2. *Non-dimensional wave number versus non-dimensional frequency.*

For example, it is supposed that the sound pressure wave propagates in liquid-filled CNT. The properties of several liquids are listed in Table 1 [44].

Table 1. *The properties of liquids [44].*

Fluid	Temperature ($^{\circ}C$)	Density, ρ_{f} (kg/m^3)	Speed, C_{f} (m/s)	Relaxation time, τ (s)
Alcohol (ethyl)	20	790	1150	10^{-12}
Turpentine	20	870	1250	10^{-12}
Water (fresh)	20	998	1481	10^{-12}

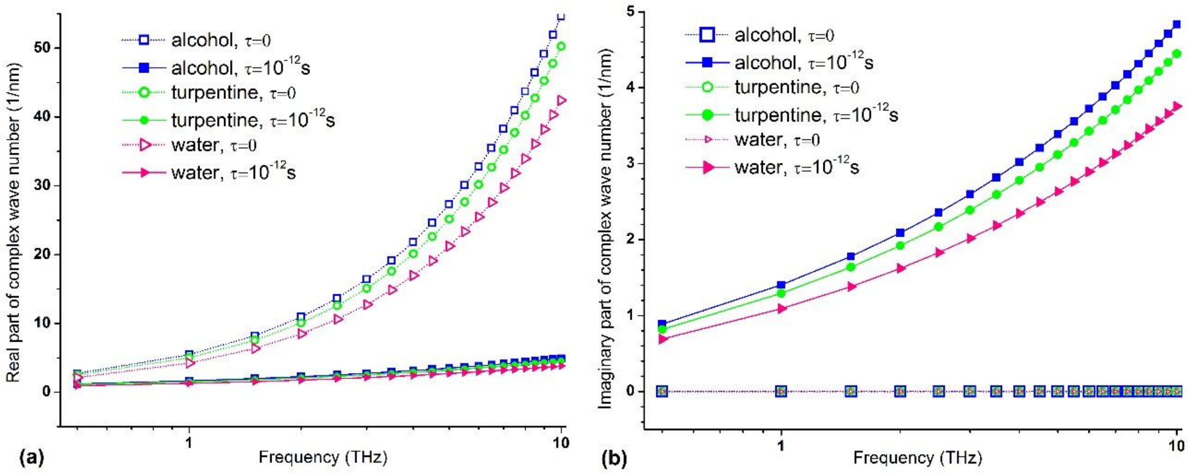

Figure 3. *(a) The real part and (b) the imaginary part of the complex wave number of modified complex Helmholtz equation versus frequency in comparison with the wave number of Helmholtz equation.*

Figures 3(a) and 3(b) show the real part and the imaginary part of the complex wave number, respectively. The inviscid–liquid (i.e. $\tau = 0$) curves in the mentioned figures are obtained from the Helmholtz equation (i.e. $\nabla^2 p + k^2 p = 0$). The viscous–liquid (i.e. $\tau \neq 0$) curves are obtained from the modified complex Helmholtz equation (i.e. $\nabla^2 p + \bar{k}^2 p = 0$).

Figure 3 shows the wave number of the Helmholtz equation (i.e. $k = \omega/C_{\mathrm{f}}$, $\tau = 0$) and the modified complex Helmholtz equation (i.e. $\bar{k} = K + i\zeta$, $\tau \neq 0$) in comparison with each other, clearly. Figure 3(a) shows that the real part of the wave number in the modified complex Helmholtz equation is much less than the wave number of the Helmholtz equation. Also, the real part of the wave number in the modified complex Helmholtz equation is approximately insensitive when increaseing the frequency, especially by increasing the characteristic impedance of the liquids. The wave number of Helmholtz equation decreases by increase-ing the characteristic impedance of the liquids. As it is shown in Figure 3(b), the imaginary part of wave number in the Helmholtz equation is zero. However, the imaginary part of wave number in the modified complex Helmholtz equation increases by increasing the frequency in THz range. This increase is due to the effect of $\omega\tau$ as in Eq. (10). Also, Figure 3(b) shows that the imaginary part of the wave number in the modified complex Helmholtz equation decreases by increase-ing the characteristic impedance (i.e. $\rho_{\mathrm{f}} C_{\mathrm{f}}$) of the viscous liquids.

The phase velocity represents the rate at which energy is transported, and is given via $c_{\mathrm{phase}} = \omega/K$. From Eq. (10), the phase velocity can be derived as

$$c_{\text{phase}} = C_{\text{f}} \left\{ \frac{2\left[1+(\omega\tau)^2\right]}{\sqrt{1+(\omega\tau)^2}+1} \right\}^{1/2} . \quad (11)$$

Figure 4 shows the non-dimensional phase velocity versus the frequency. The non-dimensional phase velocity increases by increasing the frequency. Since the phase velocity is a function of frequency, the viscous liquid in a CNT is a dispersive medium for sound wave propagation.

By neglecting the relaxation time effect, the phase velocity (Eq. (11)) is equal to C_{f}. It could be concluded that for high frequency sound wave propagation analysis of the liquid-filled CNT, the relaxation time or viscosity of the fluid must be considered through the modified complex Helmholtz equation.

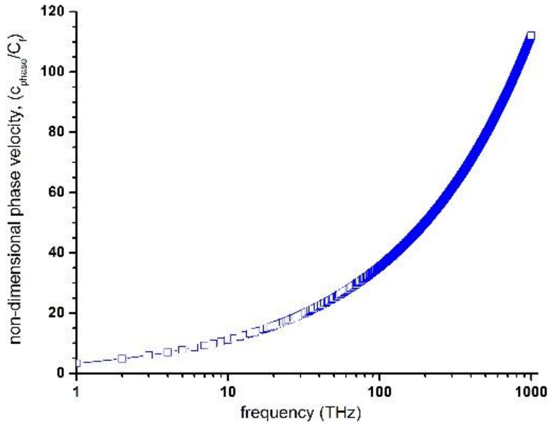

Figure 4. *Non-dimensional phase velocity versus non-dimensional frequency.*

The liquid load is exerted on the inside of the finite-length non-rigid CNT due to the sound wave propagation is $f \equiv P(R,\theta,x,t)$ obtained by the author of this paper in [28] as

$$f = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} \frac{\rho_{\text{f}}\omega^2}{\alpha_{\text{r}}^{mn} J'_m(\alpha_{\text{r}}^{mn}R) - \text{i}\bar{k}\beta J_m(\alpha_{\text{r}}^{mn}R)} J_m(\alpha_{\text{r}}^{mn}R)\cos(m\theta)\exp(\text{-i}\omega t) \times\left[W_{mn}^{+}\exp(-\text{i}\bar{\alpha}_{\text{x}}^{mn}x) + W_{mn}^{-}\exp(\text{i}\bar{\alpha}_{\text{x}}^{mn}x)\right] \quad (12)$$

where R is the shell radius and $\beta = \rho_{\text{f}} C_{\text{f}} / Z$ the acoustic specific admittance β, in which $Z = \left[p/V_{\text{r}}\right]_{r=R}$ is the acoustic impedance, where p and Vr are pressure and normal velocity on the wall, respectively. Also, α_{r}^{mn} are the radial wave numbers where m=0,1,2,… is circumferential order m and n=1,2,… is the axial half-wave number, J' is the first derivative of the first kind of Bessel function with respect to its argument, and $\bar{\alpha}_{\text{x}}^{mn} = \sqrt{\bar{k}^2 - (\alpha_{\text{r}}^{mn})^2}$ is the complex axial wave number whereas the previous researchers are considered the real axial wave number due to the neglecting of the energy loss in the liquid [29, 30] where

$$\begin{cases} \text{Re}(\bar{\alpha}_{\text{x}}^{mn}) \text{ and } \text{Im}(\bar{\alpha}_{\text{x}}^{mn}) \geq 0 & \text{for } x < 0 \\ \text{Re}(\bar{\alpha}_{\text{x}}^{mn}) \text{ and } \text{Im}(\bar{\alpha}_{\text{x}}^{mn}) < 0 & \text{for } x \geq 0 \end{cases} .$$

Thus, the complex sound pressure amplitude inside the liquid-filled finite-length non-rigid CNT per unit length, due to the sound wave propagation is as

$$\text{P} = \frac{\rho_{\text{f}}\omega^2 J_m(\alpha_{\text{r}}^{mn}R)}{\alpha_{\text{r}}^{mn} J'_m(\alpha_{\text{r}}^{mn}R) - \text{i}\bar{k}\beta J_m(\alpha_{\text{r}}^{m}} \quad (13)$$

The real part of Eq. (12) is the pressure amplitude fluctuation in the liquid and the imaginary part is the pressure amplitude associated with the viscosity force of the liquid.

Until now, the acoustic admittance of the CNT wall has been ignored by other researchers for sound wave propagation analysis in the CNT. The CNT has been considered as a perfectly rigid wall cylindrical shell with an infinite acoustical impedance. Henceforth, the second term in the denominator of Eq. (13) has not been considered in the calculations, which would cause an error in the prediction of the sound pressure acted on the liquid-filled finite-length non-rigid CNT due to the effect of the liquid load term, in the set of coupled dynamic equations in its vibrational analysis. In the following section, it is shown that considering the non-rigidity of CNT and the energy loss effect on the sound pressure in the liquid-filled finite-length CNT is essential.

3. RESULTS AND DISCUSSIONS

In this section, the complex sound pressure amplitude (Eq. (13)) inside a liquid-filled CNT with a radius of 0.678 nm and length of 29.5 nm [25], is investigated for the lowest axial half-wave mode in simply supported boundary conditions at both ends. The CNT with different wall acoustic admittances (for rigid, $\beta=0$, and for non-rigid, $\beta \neq 0$ [45]) are applied. It is supposed that the CNT is inviscid or viscous-liquid-filled.

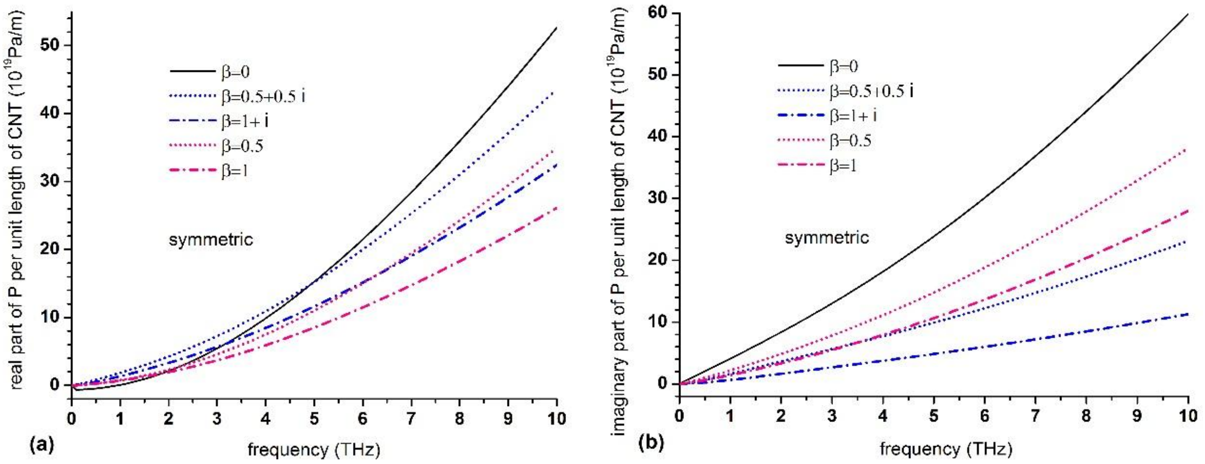

Figure 5. *(a) The pressure fluctuations and (b) the pressure associated with the viscosity force of the liquid versus frequency inside the liquid-filled rigid and non-rigid CNT in m=0.*

Figures 5(a) and 5(b) show the pressure amplitude fluctuations (the real part of the Eq. (13)) and the pressure amplitude associated with the viscosity force of the liquid (the imaginary part of the Eq. (13)) inside the liquid-filled CNT, respectively, in axi-symmetric (m=0) case for rigid ($\beta=0$) and non-rigid ($\beta \neq 0$) CNT.

Figures 6(a) and 6(b) show the pressure amplitude fluctuations and the pressure amplitude associated with the viscosity force of the liquid inside the liquid-filled CNT, respectively, in asymmetric (m=1) case for rigid ($\beta=0$) and non-rigid ($\beta \neq 0$) CNT.

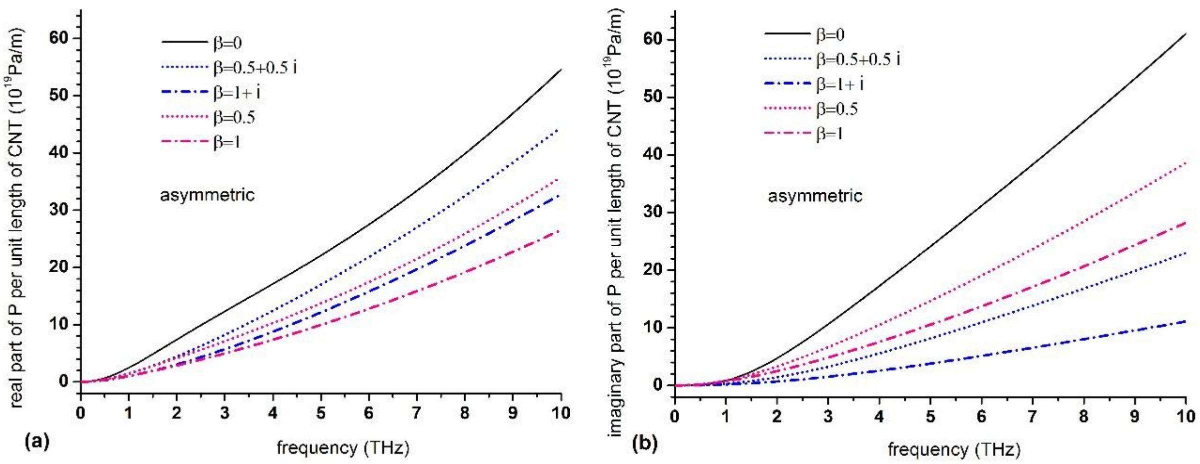

Figure 6. *(a) The pressure fluctuations and (b) the pressure associated with the viscosity force of the liquid versus frequency inside the liquid-filled rigid and non-rigid CNT in m=1.*

Figures 5 and 6 show that the non-rigidity has an important effect on the real part and the imaginary part of the pressure inside the viscous-liquid-filled CNT. As mentioned before, the relaxation time effect of the liquid and the non-rigidity of the CNT wall have not been dealt with by other researchers in vibrational analysis of the CNTs [25, 26, 29-32, 36-41] due to its complexity. Figures 5 and 6 show that the assumption of $\omega\tau << 1$ and the rigidity of the wall of the CNT would cause errors in the prediction of the pressure fluctuations and the pressure associated with the viscosity force of the liquid inside the liquid-filled CNT for both axi-symmetric and asymmetric vibrational analyses. If the non-rigidity (β) has been assumed the real number, both the real and the imaginary parts of the pressure inside the CNT decrease by increasing β. If β has been assumed the complex number, both the real and the imaginary parts of the pressure inside the CNT decrease by increasing $|\beta|$.

The results show that neglecting the non-rigidity of CNT would cause a decrease on the pressure fluctuations and the pressure associated with the viscosity force of the liquid in the liquid-filled CNT, at both axi-symmetric, and asymmetric cases.

The energy dissipation and the non-rigidity of the liquid-filled CNT have not been mentioned by the other researchers whereas their effects are essential on the liquid load that is exerted on the wall of liquid-filled CNT. Hence, the liquid load term that is used in a set of coupled dynamic equations of the CNT is not exact and would cause errors in the analysis of the sound wave propagation of the liquid-filled CNT.

3. CONCLUSION

In this paper, numerical results obtained and explained from an exact formula as obtained by the author of this paper and presented in Ref. [28] for sound pressure load term due to the existence of viscous liquid inside the finite-length non-rigid carbon nanotubes (CNTs), which is coupled with the dynamic equations of motion for the CNT. Significant points of this study include:

- The results show that the real part of the non-dimensional wave number of the modified Helmholtz equation decreases by increasing $\omega\tau$.
- The imaginary part of the non-dimensional wave number of the modified Helmholtz equation is sensitive to $\omega\tau$ and initially increases by increasing $\omega\tau$ and then, decreases by increasing $\omega\tau$.
- The results show that the assumption of $\omega\tau << 1$ would cause an error in both the real part of the complex wave number in the high frequency vibrational analysis of the liquid-filled CNT.
- The non-dimensional phase velocity increases by increasing the frequency.
- The liquid in the CNT is a dispersive medium for sound wave propagation, because the phase velocity of the wave is a function of frequency.
- The value of the pressure fluctuations and the pressure associated with the viscosity force, by considering the viscosity, would increase in both rigid and non-rigid liquid-filled CNT in both axi-symmetric and symmetric vibrational analyses.

By considering the relaxation time, the non-rigidity causes a decrease of both the pressure fluctuations, and the pressure associated with the viscosity force in the liquid-filled CNT.

REFERENCES

1. Tashakori, H., Khoshnevisan, B., Kanjouri, F., Naghavi, Z. S., (2014). “Ab Initio Study of Chirality Effects Onphonon Spectra, Mechanical and Thermal Properties of Nearly Samediameter Single Wall Carbon Nanotubes”, *Int. J. Nanosci. Nanotechnol.*, 10 (3): 163-170.

2. Rahmani, S., Mahmoudifard, M., Safi, M., (2014). “Protecting Surfaces Using One-Dimensional Nanostructures”, *Int. J. Nanosci. Nanotechnol*., 10 (1): 61-66.
3. Farhadian, N., (2011). “Transport of a Liquid Water-Methanol Mixture in a Single Wall Carbon Nanotube”, *Int. J. Nanosci. Nanotechnol*., 7 (4): 173-182.
4. Maani Miandoab, E., Tajaddodianfar, F., Nejat Pishkenari, H., Ouakad, H. M., (2015). “A nalytical Solution for the Forced Vibrations of a Nano-Resonator with Cubic Nonlinearities Using Homotopy Analysis Method”, *Int. J. Nanosci. Nanotechnol*., 11 (3): 159-166.
5. Jung, Y., Kim, T., Park, C. R., (2015). “Effect of polymer infiltration on structure and properties of carbon nanotube yarns”, *Carbon*, 88: 60–69.
6. Miao, M., (2015). “Characteristics of carbon nanotube yarn structure unveiled by acoustic wave propagation”, *Carbon*, 91: 163-170.
7. Inagaki, M., Qiu, J., Guo, Q., (2015). “Carbon foam: Preparation and application”, *Carbon*, 87: 128–152.
8. Qian, Y. J., Kong, D. Y., Liu, Y., Liu, S. M., Li, Z. B., Shao, D. S., Sun, S. M., (2014). “Improvement of sound absorption characteristics under low frequency for micro-perforated panel absorbers using super-aligned carbon nanotube arrays”, *Appl. Acoust*., 82: 23–27.
9. Haslam, M. D., Raeymaekers, B., (2014). “Aligning carbon nanotubes using bulk acoustic waves to reinforce polymer composites”, *Composites: Part B*, 60: 91–97.
10. Sayago, I., Fernández, M. J., Fontecha, J. L., Horrillo, M. C., Vera, C., Obieta, I., Bustero, I., (2012). “New sensitive layers for surface acoustic wave gas sensors based on polymer and carbon nanotube composites”, *Sens. Actuators B*, 175: 67–72.
11. Basirjafari, S., Malekfar, R., Khadem, S. E., (2012). “Low loading of carbon nanotubes to enhance acoustical properties of poly(ether)urethane foams”, *J. Appl. Phys.*, 112 (10): 104312-1-8.
12. Besseghier, A., Heireche, H., Bousahla, A. A., Tounsi, A., Benzair, A., (2015). “Nonlinear vibration properties of a zigzag single-walled carbon nanotube embedded in a polymer matrix”, *Adv. Nano Res*., 3 (1): 29-37.
13. Benguediab, S., Tounsi, A., Zidour, M., Semmah, A., (2014). “Chirality and scale rffects on mechanical buckling properties of zigzag double-walled carbon nanotubes”, *Composites Part B*, 57: 21-24.
14. Larbi Chaht, F., Kaci, A., Houari, M. S. A., Tounsi, A., Anwar, Bég, O., Mahmoud, S. R., (2015). “Bending and buckling analyses of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect”, *Steel Compos. Struct.*, 18 (2): 425-442.
15. Tounsi, A., Benguediab, S., Adda Bedia, E.A., Semmah, A., Zidour, M., (2013). “Nonlocal effects on thermal buckling properties of double-walled carbon nanotubes”, *Adv. Nano Res*., 1 (1): 1-11.
16. Ahouel, M., Houari, M. S. A., Bedia, E.A.A., Tounsi, A., (2016). “Size-dependent mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept”, *Steel Compos. Struct*., 20 (5): 963-981.
17. Zemri, A., Houari, M. S. A., Bousahla, A. A., Tounsi, A., (2015). “A mechanical response of functionally graded nanoscale beam: an assessment of a refined nonlocal shear deformation theory beam theory”, *Struct. Eng. Mech., Int. J*., 54 (4): 693-710.
18. Al-Basyouni, K. S., Tounsi, A., Mahmoud, S. R., (2015). “Size dependent bending and vibration analysis of functionally graded micro beams based on modified couple stress theory and neutral surface position”, *Compos. Struct*., 125: 621-630.
19. Wang, Y. Z., Li, F. M., Kishimoto, K., (2012). “Effects of axial load and elastic matrix on flexural wave propagation in nanotube with nonlocal Timoshenko beam model”, *J. Vib. Acoust*., 134: 031011.
20. Wang, Y. Z., Li, F. M., (2014). “Nonlinear free vibration of nanotube with small scale effects embedded in viscous matrix”, *Mech. Res. Commun*., 60: 45–51.
21. Wang, Y. Z., (2017). “Nonlinear internal resonance of double-walled carbon nanotubes under parametric excitation by nonlocal continuum theory”, *Appl. Math. Modell*., 48: 621–634.
22. Bounouara, F., Benrahou, K. H., Belkorissat, I., Tounsi, A., (2016). “A nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation”, *Steel Compos. Struct*., 20 (2): 227-249.
23. Belkorissat, I., Houari, M. S. A., Tounsi, A., Adda Bedia E. A., Mahmoud, S. R., (2015). “On vibration properties of functionally graded nano-plate using a new nonlocal refined four variable model”, *Steel Compos. Struct. Int. J*., 18 (4): 1063-1081.
24. Daneshmand, F., Rafiei, M., Mohebpour, S. R., Heshmati, M., (2013). “Stress and strain-inertia gradient elasticity in free vibration analysis of single walled carbon nanotubes with first order shear deformation shell theory”, *Appl. Math. Modell*., 37: 7983–8003.
25. Wang, L., Hu, H., Guo, W., (2006). “Validation of the non-local elastic shell model for studying longitudinal waves in single-walled carbon nanotubes”, *Nanotechnol*., 17: 1408-15.
26. Hu, Y. G., Liew, K. M., Wang, Q., He, X. Q., Yakobson, B. I., (2008). “Nonlocal shell model for elastic wave propagation in single- and double-walled carbon nanotubes”, *J. Mech. Phys. Solids*, 56: 3475-85.

27. Basirjafari, S., Malekfar, R., Khadem, S. E., (2012). "Modeling of the microstructure of carbon nanotube with two nonlocal elasticity theories", *J. Appl. Phys*., 111 (3): 034315-1-6.
28. Basirjafari, S., Khadem, S. E., Malekfar, R., (2012). "A comprehensive study of sound pressure in a finite-length fluid-filled multi-walled carbon nanotube", *Ultrasonics*, 52 (5): 655-662.
29. Natsuki, T., Ni, Q. Q., Endo, M., (2008). "Vibrational analysis of fluid-filled carbon nanotubes using the wave propagation approach", *Appl. Phys. A*, 90: 441-45.
30. Natsuki, T., Ni, Q. Q., Endo, M., (2007). "Wave propagation in single- and double-walled carbon nanotubes filled with fluids", *J. Appl. Phys*., 101 (3): 034319-1-5.
31. Liew, K. M., Wang, Q., (2007). "Analysis of wave propagation in carbon nanotubes via elastic shell theories", *Int. J. Eng. Sci*., 45: 227-41.
32. Dong, K., Wang, X., (2007). "Wave propagation in carbon nanotubes embedded in an elastic matrix", *Arch. Appl. Mech*., 77: 575-586.
33. Basirjafari, S., Khadem, S. E., Malekfar, R., (2013). "Radial breathing mode frequencies of carbon nanotubes for determination of their diameters", *Cur. Appl. Phys*., 13 (3): 599-609.
34. Basirjafari, S., Khadem, S. E., Malekfar, R., (2013). "Determination of the inner diameter of a double-walled carbon nanotube from its Raman spectra", *J. Appl. Phys*., 113 (6): 064304-1-7.
35. Basirjafari, S., Esmaeilzadeh Khadem, S., Malekfar, R., (2013). "Validation of shell theory for modeling the radial breathing mode of a single-walled carbon nanotube", *Int. J. Eng. Trans. A*, 26 (4): 447-454.
36. Basirjafari, S., Malekfar, R., Khadem, S. E., (2011). "Radial breathing mode frequency of multi-walled carbon nanotube via multiple-elastic thin shell theory", *Int. J. Nanosci. Nanotechnol*., 7 (3): 137-142.
37. Wang, C., Lai, J. C. S., (2001). "The sound radiation efficiency of finite length circular cylindrical shells under mechanical excitation II: limitations of the infinite length model", *J. Sound Vib*., 241 (5): 825-38.
38. Soltani, P., Taherian, M. M., Farshidianfar, A., (2010). "Vibration and instability of a viscous-fluid-conveying single-walled carbon nanotube embedded in a visco-elastic medium", *J. Phys. D: Appl. Phys.*, 43: 425401.
39. Yoon, J., Ru, C. Q., Mioduchowski, A., (2003). "Sound wave propagation in multiwall carbon nanotubes", *J. Appl. Phys*., 93 (8): 4801-06.
40. Heireche, H., Tounsi, A., Benzair, A., Maachou, M., Bedia, E. A. A., (2008). "Sound wave propagation in single-walled carbon nanotubes using nonlocal elasticity", *Physica E*, 40 (8): 2791-99.
41. Wang, Q., (2005). "Wave propagation in carbon nanotubes via nonlocal continuum mechanics", *J. Appl. Phys*., 98: 124301-1-6.
42. Levitsky, S., Bergman, R., Haddad, J., (2000). "Acoustic waves in thin-walled elastic tube with polymeric solution", *Ultrasonics*, 38: 857-859.
43. Elvira-Segura, L., (2000). "Acoustic wave dispersion in a cylindrical elastic tube filled with a viscous fluid", *Ultrasonics*, 37: 537-547.
44. Kinsler, L. E., Austin, R. F., Alan, B. C., James, V. S., (2000). "*Fundamentals of Acoustics*", 4th ed., John Wiley & Sons, New York.
45. Shao, W., Mechefske, C. K., (2005). "Analyses of radiation impedances of finite cylindrical ducts", *J. Sound Vib*., 286: 363-381.