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Analytical Investigation of MHD Jeffery–Hamel Nanofluid Flow in Non-Parallel Walls

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Abstract

In this paper, Homotopy perturbation method (HPM) has been applied to investigate the effect of magnetic field on Cu-water nanofluid flow in non-parallel walls. The validity of HPM solutions were verified by comparing with numerical results obtained using a fourth order Runge–Kutta method. Effects of active parameters on flow have been presented graphically. The results show that velocity in boundary layer thickness decreased with increase of Reynolds number and nanoparticle volume friction and increased with increasing Hartmann number.

Keywords: *Nanofluid; Magneto hydro dynamic; Jeffery–Hamel flow; Nonlinear Ordinary differential equation; Homotopy perturbation method.*

1. INRODUCTION

In fluid mechanics, many of the problems end up to a complicated set of nonlinear ordinary differential equation which can be solved using the different analytic method, such as perturbation method , homotopy perturbation method and variational iteration method introduced by He (1999).

The homotopy perturbation method was proposed first by He in 1998 and further developed and improved by He (2003). The method yields a very rapid convergence of the solution series in the most cases.

Sheikholeslami et al. (2011) applied HPM to investigate Hydromagnetic flow between two horizontal plates in a rotating system. Sheikholeslami et al. (2012a) studied the three-dimensional problem of steady fluid deposition on an inclined rotating disk using HPM. The flow was called Jeffery-Hamel flow after introducing the problem of the flow of fluid through a divergent channel by Jeffery (1915) and Hamel (1916). On the other hand, the term of Magneto hydro dynamic (MHD) was first introduced by Alfvén (1970). The theoretical study of (MHD) channel has been a subject of great interest due to its extensive applications in designing cooling systems with liquid metals, MHD generators, accelerators, pumps and flow meters (2002).

Fluid heating and cooling are important in many industries fields such as power, manufacturing and transportation. Effective cooling techniques are absolutely needed for cooling any sort of high energy device. Common heat transfer fluids such as water, ethylene glycol, and engine oil have limited heat transfer capabilities due to their low heat transfer properties. In contrast, metals thermal conductivities are up to three times higher than the fluids, so it is naturally desirable to combine the two substances to produce a heat transfer medium that behaves like a fluid, but has the thermal conductivity of a metal. Recently, several studies have been investigated about nanofluid. Soleimani et al. (2012) studied natural convection heat transfer in a semi-annulus enclosure filled with nanofluid. Sheikholeslami et al. (2012b) investigated the flow of nanofluid and heat transfer characteristics between two horizontal plates in a rotating system. Natural convection of a non-Newtonian copper-water nanofluid between two infinite parallel vertical flat plates is investigated by Domairry et al. (2012). Sheikholeslami et al. (2012c) studied the natural convection in a concentric annulus between a cold outer square and heated inner circular cylinders in presence of static radial magnetic field. Steady magnetohydrodynamic free convection boundary layer flow passed a vertical semi-infinite flat plate embedded in water filled with a nanofluid has been theoretically studied by Hamad et al. (2011). They found that Cu and Ag nanoparticles proved to have the highest cooling performance for this problem. Sheikholeslami et al. (2012d) performed a numerical analysis for natural convection heat transfer of Cu-water nanofluid in a cold outer circular enclosure containing a hot inner sinusoidal circular cylinder in presence of horizontal magnetic field using the Control Volume based Finite Element Method.

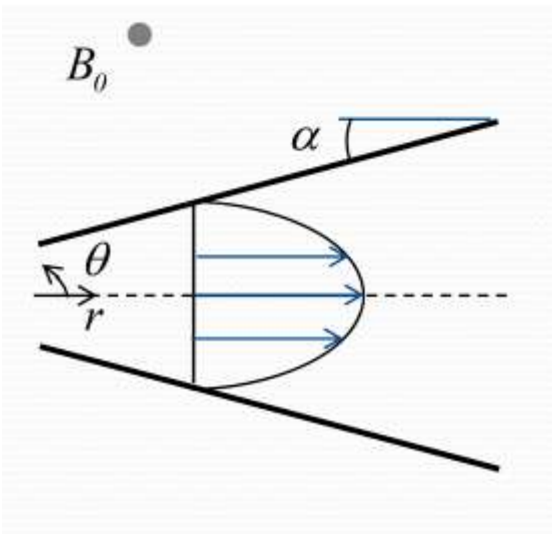


Figure1. *Figure of geometery.*

They concluded that in absence of magnetic field, enhancement ratio decreases with increasing Rayleigh number, while an opposite trend is observed in the presence of magnetic field. Recently, several papers have been published about nanofluid flow and heat transfer in presence of magnetic field (Sheikholeslami et al. (2015a, 2015b, 2015c); Sheikholeslami et al. (2016); Sheikholeslami and Ellahi (2015a, 2015b); Sheikholeslami and Rashidi (2015a, 2015b); Sheikholeslami and Shirley (2015)). In this paper, effect of magnetic field on nanofluid flow in a convergence channel has been investigated. Homotopy perturbation method (HPM) is applied to solve the governing equations. Effects of active parameters on flow are presented graphically.

2. PROBLEM FORMULATION

Consider a system of cylindrical polar coordinates (r,θ,z) in which steady two-dimensional flow of an incompressible conducting viscous fluid from a source or sink at channel walls lies in planes and intersects the z axis. Assuming purely radial motion which means that there is no change in the flow parameter along the z direction. The flow depends on r and θ and further assume that there is no magnetic field along the z-direction.

The reduced form of continuity, Navier-Stokes and Maxwell's equations are as follows (Sheikholeslami et al. (2012e)):

$$\frac{\rho_{nf}}{r}\frac{\partial(ru(r,\theta))}{\partial r}(ru(r,\theta))=0 \qquad (1)$$

$$u(r,\theta)\frac{\partial u(r,\theta)}{\partial r}=-\frac{1}{\rho_{nf}}\frac{\partial P}{\partial r} \qquad (2)$$

$$+\upsilon_{nf}\left[\begin{array}{l}\dfrac{\partial^2 u(r,\theta)}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial u(r,\theta)}{\partial r}\\ +\dfrac{1}{r^2}\dfrac{\partial^2 u(r,\theta)}{\partial\theta^2}-\dfrac{u(r,\theta)}{r^2}\end{array}\right]$$

$$-\frac{\sigma B_0^2}{\rho_{nf}r^2}u(r,\theta)$$

$$\frac{1}{\rho_{nf}r}\frac{\partial P}{\partial\theta}-\frac{2\upsilon_{nf}}{r^2}\frac{\partial u(r,\theta)}{\partial\theta}=0 \qquad (3)$$

Where B0 is the electromagnetic induction, σ_{nf} the conductivity of the fluid, $u(r)$ is the velocity along radial direction, P is the fluid pressure, υ_{nf} is the coefficient of kinematic viscosity and ρ_{nf} the fluid density.

The effective density ρ_{nf}, the effective dynamic viscosity μ_{nf} and kinematic viscosity υ_{nf} of the nanofluid are given as follows (Sheikholeslami et al. (2012e)):

$$\rho_{nf}=\rho_f(1-\phi)+\rho_s\phi,\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}}$$
$$,\upsilon_{nf}=\frac{\mu_f}{\rho_{nf}},\frac{\sigma_{nf}}{\sigma_f}=1+\frac{3\left(\frac{\sigma_s}{\sigma_f}-1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f}+2\right)-\left(\frac{\sigma_s}{\sigma_f}-1\right)\phi} \quad (4)$$

Here, ϕ is the solid volume fraction. Considering $u_\theta=0$ for purely radial flow, one can define the velocity parameter as:

$$f(\theta)=ru(r) \quad (5)$$

Introducing the $\eta=\frac{\theta}{\alpha}$ as the dimensionless degree, the dimensionless form of the velocity parameter can be obtained by dividing that to its maximum value as:

$$f(\eta)=\frac{f(\theta)}{f_{\max}} \quad (6)$$

ubstituting Eq.5 into Eqs.2 and 3, and eliminating P, one can obtain the ordinary differential equation for the normalized function profile as below (Anwari et al. (2005).):

$$f'''(\eta)+2\alpha\,\mathrm{Re}\,A^*(1-\phi)^{2.5}f(\eta)f'(\eta)$$
$$+(4-(1-\phi)^{1.25}Ha)\alpha^2f'(\eta)=0 \quad (7)$$

Where A^* is a parameter Reynolds number and Hartmann number based on the electromagnetic parameter are introduced as following form:

$$A^*=(1-\phi)+\frac{\rho_s}{\rho_f}\phi,B^*=\frac{\sigma_{nf}}{\sigma_f} \quad (8)$$

$$\mathrm{Re}=\frac{f_{\max}\alpha}{\upsilon_f}=\frac{U_{\max}r\alpha}{\upsilon_f} \quad (9)$$

$$Ha=B_0\sqrt{\frac{\sigma_{nf}}{\rho_f\upsilon_f}} \quad (10)$$

With the following reduced form of boundary conditions:

$$f(0)=1,f'(0)=0,f(1)=0 \quad (11)$$

3. HPM

To illustrate the basic ideas of this method, we consider the following equation:

$$A(u)-f(r)=0,\ r\in\Omega \quad (12)$$

With the boundary condition of:

$$B\left(u,\frac{\partial u}{\partial n}\right)=0,\ r\in\Gamma \quad (13)$$

Where *A* is a general differential operator, *B* a boundary operator, *f (r)* a known analytical function and *Γ* is the boundary of the domain *Ω*.

A can be divided into two parts which are *L* and *N*, where *L* is linear and *N* is nonlinear. Eq. (12) can therefore be rewritten as follows:

$$L(u)+N(u)-f(r)=0,r\in\Omega \quad (14)$$

Homotopy perturbation structure is shown as follows:

$$H(\nu,p)=(1-p)\left[L(\nu)-L(u_0)\right]$$
$$+p\left[A(\nu)-f(r)\right]=0 \quad (15)$$

Where,

$$\nu(r,p):\ \Omega\times[0,1]\to R \quad (16)$$

In Eq. (16), $p \in [0,1]$ is an embedding parameter and u_0 is the first approximation that satisfies the boundary condition. We can assume that the solution of Eq. (15) can be written as a power series in *p*, as follows:

$$\nu = \nu_0 + p\nu_1 + p^2\nu_2 + ... \tag{17}$$

And the best approximation for solution is:

$$u = \lim_{p \to 1} \nu = \nu_0 + \nu_1 + \nu_2 + ... \tag{18}$$

4. IMPLEMENTATION OF HPM

By applying HPM on Eq. (15), the following equation can be obtained:

$$\begin{aligned}&(1-p)(f''' - f_0''') + H_1(P)(f'''(\eta)\\&+2\alpha \,\mathrm{Re}.A^*(1-\phi)^{2.5} f(\eta) f'(\eta)\\&+(4-(1-\phi)^{1.25} Ha)\alpha^2 f'(\eta)) = 0\end{aligned} \tag{19}$$

We consider f as follows:

$$f(\eta) = f_0(\eta) + f_1(\eta) + ... = \sum_{i=0}^{n} f_i(\eta) \tag{20}$$

With substituting f from Eq. (20) to Eq. (19) and some simplification and rearranging based on powers of *p*–terms, we have:

$$\begin{aligned}&p^0: \ f''' = 0,\\&\quad f_0(0) = 1, f_0'(0) = 0, f_0(1) = 0.\end{aligned} \tag{21}$$

$$\begin{aligned}&p^1: \ f_1''' + 2\alpha \,\mathrm{Re} A^*(1-\phi)^{2.5} f_0' f_0 - f_0'''\\&\quad -\alpha^2 Ha(1-\phi)^{1.25} f_0' + 4\alpha^2 f_0' = 0,\\&\quad f_1(0) = 0, f_1'(0) = 0, f_1(1) = 0.\end{aligned} \tag{22}$$

Solving Eqs. (21) and (22) with boundary conditions , we have:

$$f_0(\eta) = -\eta^2 + 1, \tag{23}$$

$$\begin{aligned}f_1(\eta) = &(-0.3333\alpha \,\mathrm{Re} A^*(1-\phi)^{2.5}\eta^6\\&-0.0833\alpha^2 Ha\,\eta^4 + 0.3333\alpha^2\eta^4\\&+0.1667\alpha \,\mathrm{Re} A^*(1-\phi)^{2.5}\eta^4\\&-0.1333\alpha \,\mathrm{Re} A^*(1-\phi)^{2.5}\eta^2\\&+0.0833\alpha.Ha.(1-\phi)^{1.25}\eta^2 - 0.3333\alpha^2\eta^2)\end{aligned} \tag{24}$$

The solution of this equation, when $p \to 1$, will be as follows:

$$f(\eta) = f_0(y) + f_1(y) + f_2(y) + ... \tag{25}$$

Table1. *Thermo physical properties of water and nanoparticle [14]*

	Pure water	Copper
$\rho(kg/m^3)$	*997.1*	*8933*
$C_p(j/kgk)$	*4179*	*385*
$k(W/m.k)$	*0.613*	*401*
$\beta \times 10^5(K^{-1})$	*21*	*1.67*

5. RESULTS AND DISCUSSION

In this study, MHD Jeffery–Hamel flow in nanofluid filled non-parallel walls is investigated analytically utilizing homotopy perturbation method. The percentage of error is introduced as fallow:

$$\%Error = \left| \frac{f(\eta)_{NM} - f(\eta)_{HPM}}{f(\eta)_{NM}} \right| \times 100 \tag{26}$$

Figure. 2 shows that %Error of f at different step versus η .

Also, comparison between the Numerical results and HPM solution of velocity including different values of active parameters is shown in this figure. As seen in this figure homotopy-perturbation method is converged in step 8 and error is minimized. The profile of error versus η has fluctuated behavior and maximum value of error is obtained at centerline of channel. The results were well matched with the results carried out by numerical solution (Rung-kutta) as shown in Table 2 and Figure1(c).

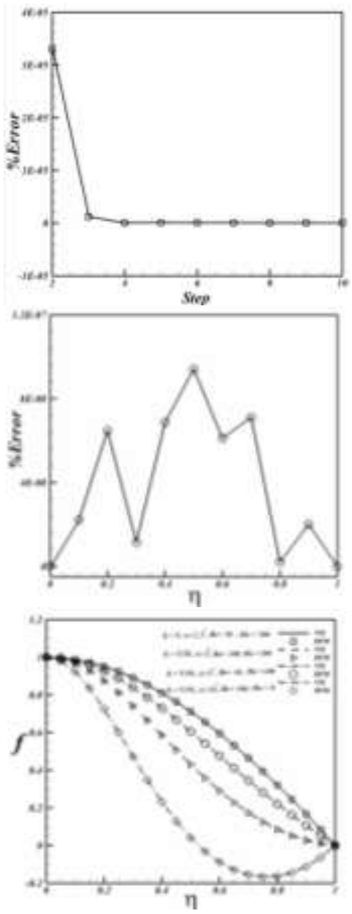


Figure2. *%Error of f at different step; versus η when $\phi = 0.06, Ha = 5, Re = 5, \alpha = 2.5^{\circ}$; Comparison between the Numerical results and HPM solution of velocity including different values of active parameters*

Figure3 shows that effect of nanoparticle volume fraction on velocity profiles. This figure indicates that increasing nanoparticle volume fraction leads to decrease the velocity boundary layer thickness. Besides it can be seen that for higher values of Reynolds number this decrement is more sensible. Effect of Hartmann number, Reynolds number and Angle of the channel on velocity profiles is shown in Figure. 4. It is worth to mention that the Reynolds number indicates the relative significance of the inertia effect compared to the viscous effect.

Table2. *Comparison between the Numerical results and HPM solution of velocity when Cu-Water $\phi = 0.06$ and (a) $Ha = 10, \alpha = 5°, Re = 5$;(b) $Ha = 20, \alpha = 2.5°, Re = 50$* .

η	%Error	
	(a)	(b)
0	0	9.99201E-14
0.1	1.12E-07	1.10681E-07
0.2	7.06E-08	4.72901E-07
0.3	1.27E-07	8.78684E-07
0.4	1.56E-07	1.23221E-06
0.5	1.83E-07	1.50356E-06
0.6	5.57E-07	1.88496E-06
0.7	5.45E-07	2.25485E-06
0.8	5.97E-07	2.78542E-06
0.9	1.32E-06	4.85791E-07
1	0	0

Thus, velocity profile decreases with increasing Re and in turn increasing Re leads to increase in the magnitude of the skin friction coefficient.

Generally, when the magnetic field is imposed on the channel, the velocity field is suppressed owing to the retarding effect of the Lorenz force.

Thus, the presence of magnetic field increases the momentum boundary layer thickness. The results show moderate increase in the velocity with increasing Hartmann numbers at small angle($\alpha = 2.5°$) and difference between velocity profiles are more noticeable at greater angles. Backflow is excluded in converging channels but it may occur for large Reynolds numbers in diverging channels.

For specified opening angle, after a critical Reynolds number, we observe that separation and backflow is started. By increasing Hartmann number this phenomenon is eliminated and higher values of Reynolds number needs greater magnetic field to eliminate the back flow.

(a)

(b)

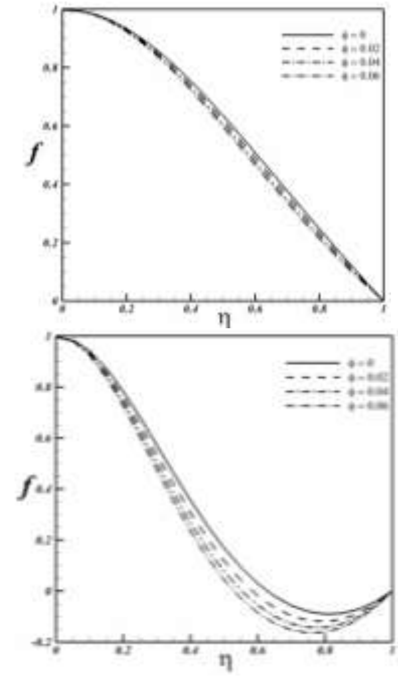
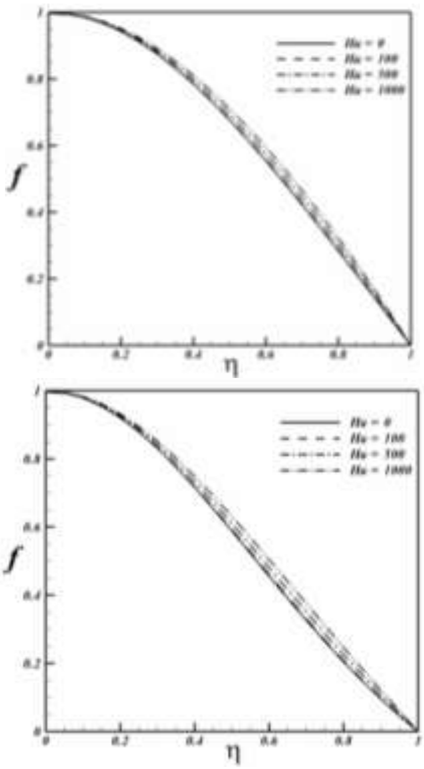


Figure3. *Effect of nanoparticle volume fraction on velocity profiles for Cu-Water when (a)* $Ha = 50, Re = 50$ *;(b)* $Ha = 50, Re = 100$.

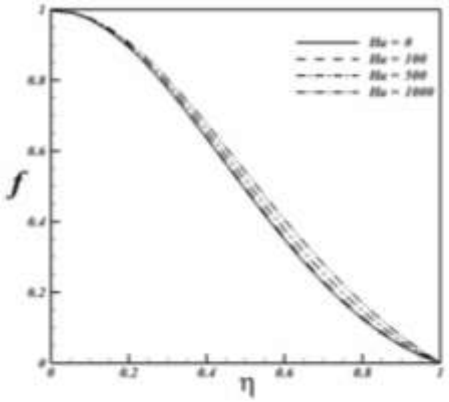
$\alpha = 2.5^{\circ}$

Re=50

Re=100



Re=200



$\alpha = 10^{\circ}$

Re=50

Re=100

Re=200

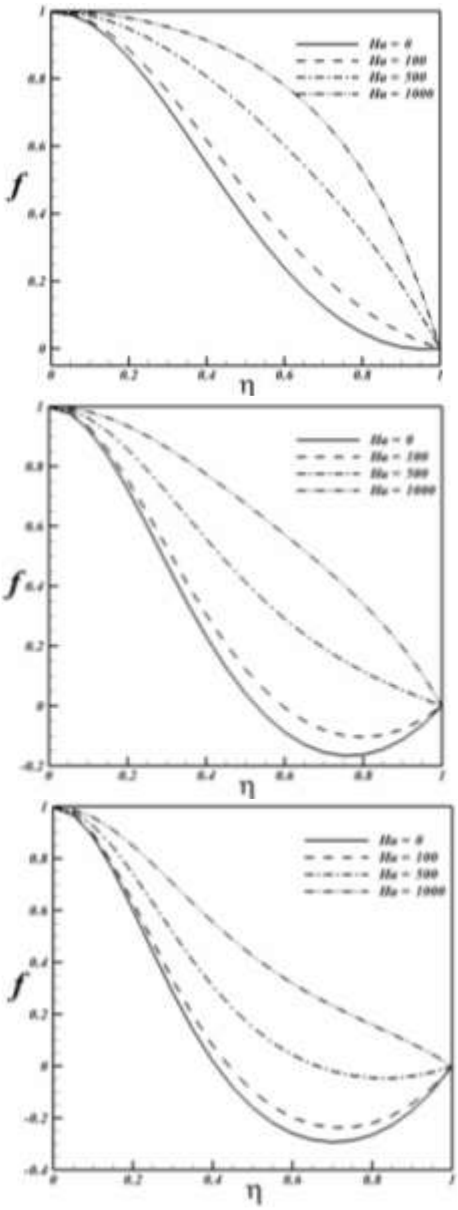


Figure4. *Effect of Hartmann number, Reynolds number and Angle of the channel on velocity profiles for Cu-Water* $\phi = 0.06$

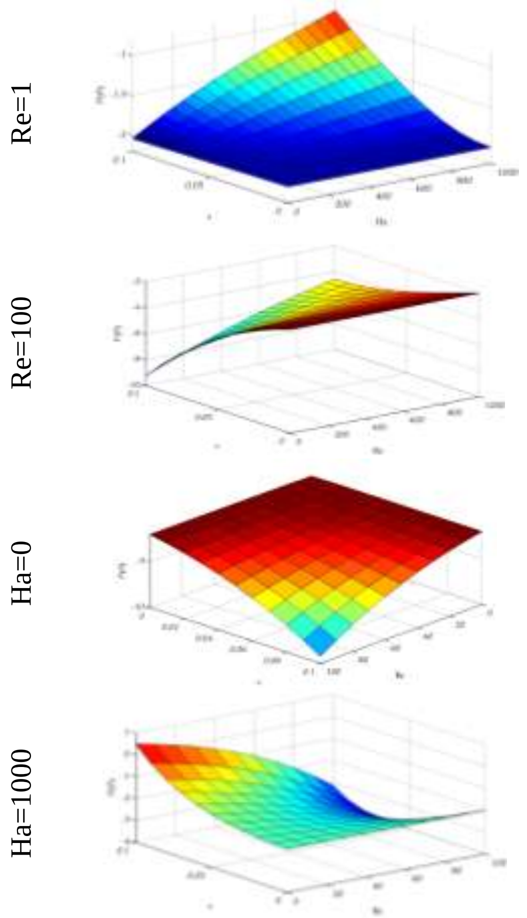


***Figure5.** Effects of Angle of the channel, Reynolds number and Hartmann number on Skin friction coefficient* $f''(0)$ *when* $\phi = 0.06$.

Effects of Angle of the channel, Reynolds number and Hartmann number on Skin friction coefficient $\left(f''(0)\right)$ is shown in Figure. 5. Skin friction coefficient is an increasing function of Reynolds number, opening angle and nanoparticle volume friction but a decreasing function of Hartmann number.

6. CONCLUSION

In this paper, homotopy perturbation method is used to solve the problem of magneto hydrodynamic Jeffery-Hamel nanofluid flow. It can be found that homotopy perturbation method is a powerful approach for solving this problem and also it is observed that there is a good agreement between the present and numerical result. The results indicate that velocity boundary layer thickness decreases with increasing Reynolds number and nanoparticle volume friction, while it rises with increasing Hartmann number. Also, it can be found that Hartmann number is a key factor for back flow and higher values of Reynolds number need more magnetic fields to eliminate the back flow.

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