Short Communication

Convection Heat Transfer Modeling of Nano- fluid Tio_2 Using Different Viscosity Theories

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(Received: 24 Oct. 2012 and accepted: 23 Nov. 2014)

Abstract:

In this paper, the effects of adding nanoparticles including Tio_2 to a fluid media for purpose of free convection heat transfer improvement were analyzed. The free convection was assumed to be in laminar flow regime and the solutions and calculations were all done by the integral method. Water, as a Newtonian fluid, was considered the base fluid (water) and all the thermo physical properties of the nano-fluids have been considered unvarying. According to the calculations performed and all depicted graphs, one can thoroughly consider that adding nanoparticles to the fluid generally causes increment and development of heat transfer coefficient. Increasing nanoparticles concentration increases the heat transfer rate. The heat transfer increment is also dependent on the nanoparticles thermal conductivity and the viscosity theory used. In this study, four different kinds of theories were used for the nano-fluids viscosity calculations. The effects of viscosity on the nano-fluids thermal conductivity can be obviously seen. All the calculations have been done for the concentrations lower than 4%.

Keywords: *Nano- fluids, Heat transfer enhancement, Natural convection.*

1. INRODUCTION

Optimization of heat transfer in the direction of achieving more efficiency needs concentration on the facilities scale-down and also increasing the heat transfer rate. The conductivity of some metallic particles, metallic oxides and nanotubes is much higher than the conductivity of liquids. According to the novel ideas, adding very tiny particles in to liquids (which makes nano-fluids) may develop the heat transfer rate remarkably [1, 2]. Natural convection heat transfer is an important phenomenon in engineering system due to its wide application in electronics, cooling, heat exchanger and double pane windows [3, 4]. It is obvious that using nano-fluids and researches about that subject have been popularized during the recent years. The investigations about forced convection have been mostly experimental and the studies about the free convection heat transfer as experimental and theoretical have well grown simultaneously [5]. The studies and experiments show that the forced convection heat transfer of nano-fluids is dependent on Reynolds number [6] although beside all the experimental activities about this subject, lack of sufficient information is absolutely evident [7].

Feasibility and efficacy of using various nano-fluids for convection heat transfer enhancement have been experimentally explored mostly for forced convection hear transfer [8, 9]. Different experiments and studies have been done on the improvement of free convection heat transfer and the results have been published by several papers [10, 11]. The nanoparticles concentrations highly affects the convection heat transfer improvement, which is associated with increasing of heat transfer coefficient and the nanoparticles Nusselt number [12]. The experimental studies on TiO_2 nano-fluids have been done by Ding et al. [13, 14]. Also, some other experimental regarding the obtained results by the performe experimental works, it is crystal clear that the heat transfer coefficient (h) is directly dependent on k/δ_t where k is conductivity and δ_t is the thickness of thermal boundary layer. One of the main reasons for increasing the heat transfer rate of nano-fluids comparing to the base fluid individually, is raising the heat transfer coefficient by adding nanoparticles. Existence of numerous papers about nano-fluids verifies the declaration [15, 16]. The amount of enhancement is dependent on the particle shape, particle type, particle size, particle Concentration and kind of base fluid [17]. In the recent works, one can apparently understand that in addition to all the mentioned factors, viscosity has an effect on Heat transfer improvement and plays nanoparticles conductivity a significant role on heat transfer [18, 19].

Nomenclature

Pr	Prandtl number
$Cp[J\,kg^{-1}\,K^{-1}]$	heat capacity
$k[Wm^{-1}\,K^{-1}]$	thermal conductivity
$\mu[kg/ms]$	dynamic viscosity
$\vartheta[m^2/S]$	kinematic viscosity
$\rho_f[kg/m^3]$	fluid density
UHF	uniform heat flux
UWT	uniform wall temperature
Gr	Grashof Number
Gr^*_{bf}	modified Grashof number
Nu	Nusselt number
$g\,[m\,s^{-2}]$	gravitational acceleration
$h[W\,m^{-2}\,k^{-1}]$	local heat transfer coefficient
Ra	Rayleigh number
$T[K]$	dimensional temperature
$U\,[m\,s^{-1}]$	dimensional x
Greek symbols	
$\delta_t\,[m]$	thermal boundary layer thicknesses
$\delta\,[m]$	dynamical boundary layer thicknesses
φ	volume fraction
$\beta[K^{-1}]$	volumetric expansion coefifcien.
ε	the heat transfer performance
a	Constant
Ω	ratio of the thermal boundary layer thickness (δ_T) to that of the dynamical(δ)
Subscripts	
p	Nanoparticle
b_f	base lfui.
r	Nano fluid/base fluid ratio
Δ	Thermal to velocity layer thickness ratio

To investigate the effect of viscosity on heat transfer coefficient, four different models have been used in this paper. The modeling results have been compared with the similar experimental results. In principle, increasing of random movement of nanoparticles causes' thermal conductivity enhancement in nano-fluids in this study, an appropriate mechanical model has been chosen which is solved by integral solution methods. The results may be used for both states of UHF and UWT which can be applied for both laminar and turbulent regimes [20].

2. MECHANICAL MODEL

The applied model is a vertical plane which on free convection in laminar conditions occurs. According to Figure 1, two kinds of boundary conditions including uniform heat flux (UHF) and uniform wall temperature (UWT) may be used for this model.

In reference to what was stated in the introduction, all fluid properties are assumed constant except viscosity. Writing the continuity, momentum and energy equations in directions of x and y for steady-state conditions will help us to obtain the target equations. Also the simultaneous solution of the following integrals, considering a four-power polynomial profile for velocity and temperature, leads to the following equations.

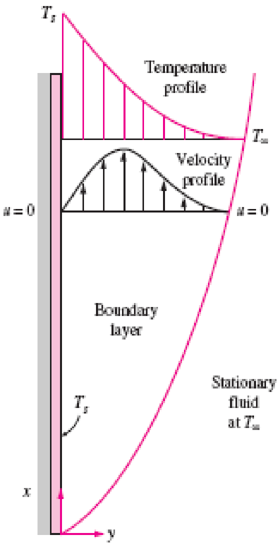

Figure 1. *Temperature and velocity distribution of free convection boundary layer*

Continuity equation:

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0 \qquad (1)$$

Momentum equation:

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+V\frac{\partial V}{\partial y}=g\beta(T_W-T_\infty)+v\frac{\partial^2 U}{\partial y^2} \qquad (2)$$

Energy equation:

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial x}+V\frac{\partial T}{\partial y}=\frac{\vartheta}{Pr}\frac{\partial^2 T}{\partial y^2} \qquad (3)$$

Using the integral solution and considering

$$U=\frac{g\beta\rho_\omega\Omega\partial^3}{12\lambda v}[-\eta^4+3\eta^3-3\eta^2+\eta]$$

$$\theta=(T-T_\infty)=\frac{\rho_W\Omega\partial}{2\lambda}(-\eta_T{}^4+2\eta_T{}^3-2\eta_T+1) \qquad (4)$$

the mentioned profile for velocity and temperature, we would have:

$$\eta={}^{y}/_{\eta}\leq 1 \text{ and } \eta_T={}^{y}/_{\eta_T}\leq 1$$

The relation between thermal boundary layer thickness and hydrodynamic boundary layer thickness may be shown as follows:

$$\partial_{T(X,T)}=\Omega\partial(X,T) \qquad (5)$$

Putting equation 4 into the following integrals and solving these two integrals simultaneously,

$$\frac{\partial}{\partial t}\int_0^{\delta}U dy+\frac{\partial}{\partial x}\int_0^{\delta_{nf}}U^2\,dy= g\beta_{nf}\int_0^{(\Omega\delta)_{nf}}(T-T_\infty)\,dy-\vartheta_{nf}=0 \qquad (6a)$$

$$\frac{\partial}{\partial t}\int_0^{\Omega\delta}\theta dy+\frac{\partial}{\partial x}\int_0^{(\Delta\delta)_{nf}}(T-T_\infty)U dy= \frac{\vartheta_{nf}}{Pr_{nf}}\left(\frac{\partial T}{\partial y}\right)_y=0 \qquad (6b)$$

Ω may be obtained as equation 7. (Considering ∂/∂t=0 for steady-state conditions and ln Pr = k)

$$\Omega=1.576\times10^{-4}K^4-4.227\times10^{-3} +4.282\times10^{-2}k^2-0.1961k+0.901 \qquad (7)$$

According to what is observed, Ω is a function of Pr and Pr is a function of nano-fluid viscosity, so the amount of Ω is noticeably changed with Pr.

In this paper, different models of viscosity have been applied. Different parameters affect nano-fluid viscosity which from, the nanoparticles shape and the shear exerted into fluid are of the most significance to improve heat transfer [21]. Regarding to the experimental and practical researches, investigators have found that viscosity is dependent on the method of preparation of nanoparticle as well [22]. Nanoparticles, because of very fine structure, make a stable and homogeneous state when solving in the base fluid. This, causes the nanofluids are considered similar to the base fluids as mono-phase. Since, temperature raising causes increase in the nanoparticles mean diameter

[23], temperature is taken constant in this paper for precise solution of the integrals. The mean diameter of nanoparticles is assumed to be 32-50 nm and the calculations have been done for UHF and UWT boundary conditions. After solving the integrals, the parameters are substituted in Nu and the effects of Ω changes on Nu are investigated.

The Maxwell Wasp model is chosen for Nu calculations, which has agreement with the experimental results [24]. The average Nusselt number along the wall may be obtained by the following equation [19].

$$\overline{Nu_{nf}} = \frac{\overline{h_{nf}}L}{k_{bf}} \quad (8)$$

Meanwhile, the average Nusselt numbers are different for UHF and UWT boundary conditions. For UWT boundary condition [19]:

$$\overline{Nu_{nf}} = \frac{4\sqrt{5}}{3\Delta_{nf}}\left[\frac{\beta_r k_r^4}{378\nu_r^2(9\Delta_{nf}-5)}Gr_{bf}\right]^{\frac{1}{4}} \quad (9)$$

Also, for UHF boundary condition [19]:

$$\overline{Nu_{nf}^{*}} = \frac{6}{5}\left[\frac{2\beta_r k_r^4}{27\nu_r^2(9\Delta_{nf}-5)\Delta_{nf}^4}Gr_{bf}^{*}\right]^{\frac{1}{5}} \quad (10)$$

Gr_{bf*} is the modified Grashof number [19]:

$$Gr_{bf}^{*} = \frac{g\beta\rho_w L^4}{K_{bf}\vartheta_{bf}^{2}} \quad (11)$$

The boundary-layer change rate is

$$f(\Delta) = \frac{9\Delta_{bf}^5 - 5\Delta_{bf}^4}{9\Delta_{nf}^5 - 5\Delta_{nf}^4} \quad (12)$$

The ε function (expressing the heat transfter performance) is [19]:

$$\varepsilon(\%) = 100\left[\left(\frac{\beta_r k_r^4}{\nu_r^2}f(\Delta)\right)^{\alpha} - 1\right] \quad (13)$$

α varies with different boundary conditions, α=1/5 for UHF and α=1/4 for UWT. Also, the r index states the nanofluids/base fluids ratio.

The nanoparticles concentration is shown by subscript f and we use the available relations for obtaining viscosity, constant temperature specific heat and β:

$$\rho_{nf} = (1-\varphi)\rho_{nf} + \varphi\rho_p \quad (14)$$

$$(C_p)_{nf} = (1-\varphi)(C_p)_{bf} + \varphi(C_p)_p \quad (15$$

$$(\rho\beta_{nf}) = (1-\varphi)(\rho\beta)_{bf} + \varphi(\rho\beta)_p \quad (16)$$

For the calculation of nano-fluid conductivity, the Wasp model has been used [24]:

$$K_r = \frac{K_p + 2K_{bf} - 2\varphi(K_{bf} - K_p)}{K_p + 2K_{bf} + \varphi(K_{bf} - K_p)} \quad (17)$$

As stated, for investigating the effects of viscosity, four viscosity models have been used. The first model which is introduced for nano-fluids, is the Brinkman model [25]:

$$\mu_r = \frac{\mu_{nf}}{\mu_{bf}} = \frac{1}{(1-\varphi)^{\frac{5}{2}}} \quad (18)$$

The second model is the model which was presented by Einstein`s [26]:

Table 1. Average NUSSELT NUMBER of nano-fluids (TiO_2/ water) Model (I)

	NUUHF		
Gr	10^3	10^4	10^5
φ=1	4.204	6.662	10.559
φ=2	4.253	6.742	10.685
φ=3	4.319	6.845	10.848
φ=4	4.358	6.906	10.946
NUUWT			
φ=1	5.943	10.568	18.792
φ=2	6.033	10.729	19.079
φ=3	6.149	10.935	19.446
φ=4	6.219	11.059	19.666

$\mu_r = \frac{\mu_{nf}}{\mu_{bf}} = (1+2.5\varphi)$For $\varphi < 0.05$ (19)

The third model is presented by considering the Brownian movement [27]:

$$\mu_r = \frac{\mu_{nf}}{\mu_{bf}} = 1 + 2.5\varphi + 6.17\varphi^2 \quad (20)$$

And the fourth model, which was presented by Pakand & Cho, is [28]:

$$\mu_r = \frac{\mu_{nf}}{\mu_{bf}} = 1 + 3.911\varphi + 533.9\varphi^2 \quad (21)$$

Table 2. *Average NUSSELT NUMBER of nano-fluids (Tio₂/ water) Model (II)*

NU_{UHF}			
Gr	10^3	10^4	10^5
$\varphi = 1$	4.203	6.661	10.557
$\varphi = 2$	4.249	6.735	10.675
$\varphi = 3$	4.310	6.831	10.826
$\varphi = 4$	4.354	6.8999	10.935
NU_{UWT}			
$\varphi = 1$	5.941	10.565	18.787
$\varphi = 2$	6.027	10.717	19.058
$\varphi = 3$	6.134	10.907	19.396
$\varphi = 4$	6.256	11.125	19.783

The calculations have been done for all of the mentioned viscosity models and for taking a better and more logical conclusion, temperature is taken around 22° C, the Nano particles mean diameter is considered as 32-50 nm and the maximum concentration is assumed to be 4%. Number for TiO_2 for both UHF and UWT boundary conditions.

The results show the Nusselt changes with viscosity models, particles size, Grashof number for TiO_2 for both UHF and UWT boundary conditions.

3. RESULTS AND DISCUSSION

Considering the assumptions mentioned in the previous part, the properties of Tio_2 were obtained. According to different Gr numbers, volume fractions and viscosity models, the UHF and UWT Nusselt numbers were obtained (Tables 1-4 for Tio_2).

Table 3. *Average NUSSELT NUMBER of nano-fluids (Tio₂/ water) Model (III)*

NU_{UHF}			
Gr	10^3	10^4	10^5
$\varphi = 1$	4.203	6.661	10.557
$\varphi = 2$	4.274	6.774	10.737
$\varphi = 3$	4.312	6.834	10.831
$\varphi = 4$	4.358	6.906	10.946
NU_{UWT}			
$\varphi = 1$	5.889	10.491	18.656
$\varphi = 2$	5.986	10.644	18.928
$\varphi = 3$	6.094	10.837	19.271
$\varphi = 4$	6.182	10.993	19.5495

Table 4. *Average NUSSELT NUMBER of nano fluids (TiO_2/ water) Model (IV)*

NU_{UHF}			
Gr	10^3	10^4	10^5
$\varphi = 1$	3.982	6.311	10.003
$\varphi = 2$	3.835	6.078	9.634
$\varphi = 3$	3.713	5.885	9.327
$\varphi = 4$	3.620	5.738	9.094
NU_{UWT}			
$\varphi = 1$	5.510	9.798	17.424
$\varphi = 2$	5.213	9.271	16.486
$\varphi = 3$	4.953	8.807	15.662
$\varphi = 4$	4.7598	8.464	15.052

Table 5. *Properties of TiO_2*

	TiO_2	φ=1	φ=2	φ=3	φ=4
pr	4250	1029.6	1062.158	1094.687	1127.216
Cp	686.2	4144.072	4109.414	4074.216	4039.288
K	8.95	0.628	0.664	0.659	0.675
K_r	-	1.025	1.04998	1.076	1.102
β	2.4	0.2023	0.1951	0.188	0.182
β_r	-	0.963	0.992	0.897	0.866

As can be seen in the tables, increasing Gr number causes of raise in Nusselt number and consequently increases the convection heat transfer. Increasing the volume fraction shows an ascending trend for all conditions, as a result, the volume fraction changes of nanoparticles up to 4%, have a direct effect on heat transfer efficiency. Also, for all conditions, the level of Nusselt is higher for the UWT state compared with the UHF state. According to the experimental results, the Nusselt numbers obtained by modeling match the experiments when Gr and volume fraction increase.

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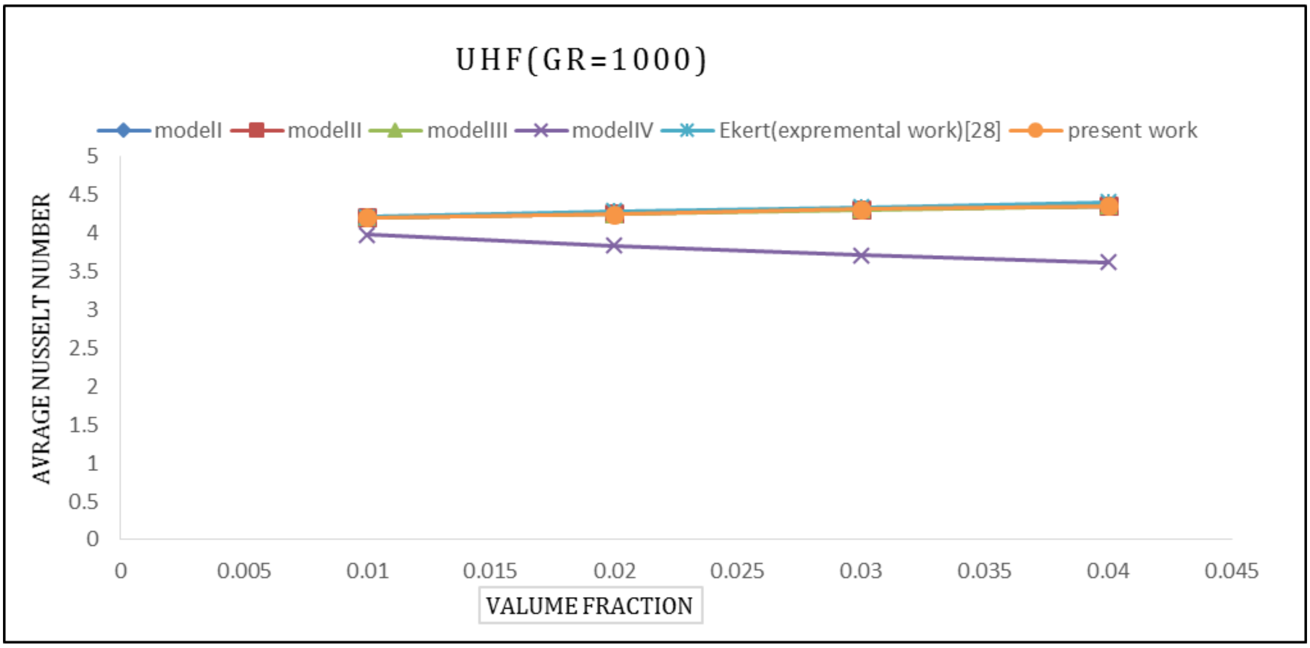

Figure 2. *Nusselt number versus volume fraction for different viscosity models at UHF (nanoparticle: TiO_2/ $Gr=10^3$)*

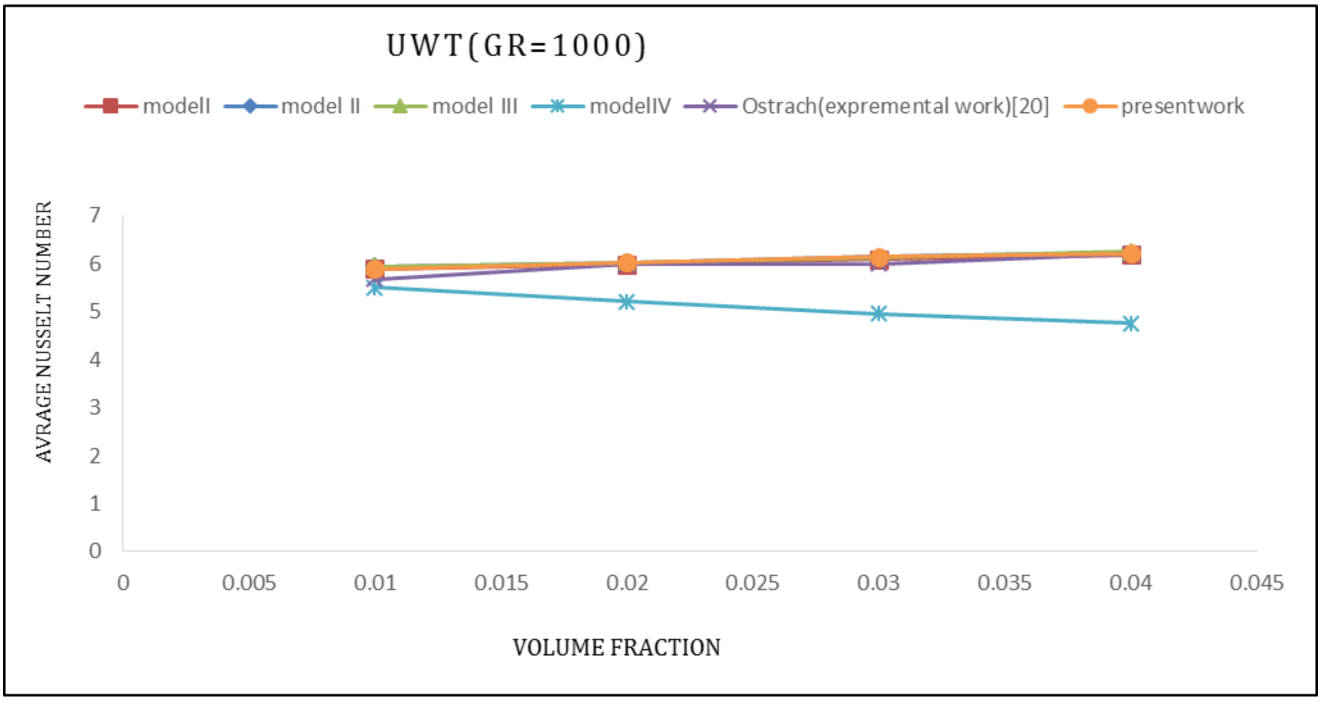

Figure 3. *Nusselt number versus volume fraction for different viscosity models at UWT (nanoparticle TiO_2/ $Gr=10^3$)*

volume fraction of the nanoparticles increases. According to the data shown on the graphs, the models numbers 1, 2 and 3 have a proper agreement to each other, but the fourth model (the Pakand & Cho model) has a different trend. So, the comparison of the results with the experimental results [12] confirms this statement that modeling by the Pakand viscosity model makes prediction deviation, but the first, second and third considered viscosity models have conformity with the experimental results.

4. CONCLUSION

The Pakand viscosity model predicts a higher increase in viscosity when adding more nanoparticles and this behavior, causes nonconformity with the other models. To predict free convection heat transfer of nano-fluids (containing metal or metal oxide particles), the Brinkman model, the Einstein model and the Brownian movement model are proposed to be applied for précised calculations. The free convection heat transfer coefficients of Tio_2 nano-fluid enhance with increasing the nanoparticles volume fraction up to 4%.

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